Theoretical Analysis of Nominal Stress in Rim of Planet Gear

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Abstract

A theoretical method of obtaining the nominal stress in the rim of a planet gear with both a thin rim and the clearance between the rim and the gear shaft is shown by using an equivalent ring. In the analysis, the boundary position between the region where the rim is in contact with the shaft and that where they are out of contact with each other is assumed because the boundary position depends on the tooth load and the clearance in the case of the constant rim thickness. Then after the bending moment in the one region is compared with that in the other one at the assumed boundary position, the true boundary position where the bending moment in the one region is equal to the other one is determined. Using the true boundary position obtained, the bending moment and the circumferential force in the rim are obtained in order to obtain the nominal bending and circumferential stresses in the rim of the planet gear. Further, effects of tooth load and the clearance on the stress in the planet gear are investigated by the method obtained.

1. Introduction

As planetary gears are compact and have great ability of power transmission, they are widely used as mechanical reducer in many branches of industry. In planetary gears, the rim of a planet gear becomes thinner and the radial clearance is made between the rim of a planet gear and the gear shaft frequently both to distribute the equal load on each planet gear and to decrease the inertia force of a planet gear. Fillet stress in the planet gear with a thin rim is different from that in the gear with a thick rim because of the effect of the stress due to the deformation of the rim. Consequently the fillet stress in the planet gear should be made clear to estimate the bending strength of the planet gear. The fillet stress in the planet gear with a thin rim seems to be composed of both the stress due to bending the tooth and that due to bending the rim. Further each stress mentioned above seems to be obtained by multiplying the nominal stress by the stress concentration factor. Whereas concerning the stress concentration of the gear with a thin rim, an approximate equation has been obtained by Chong, et al.[1] from the result of the finite element method. Moreover, the nominal stress due to bending the tooth can be obtained easily. However as a region of the contact between the rim and the gear shaft depends on both the tooth load and the clearance between the rim and the gear shaft in the case of the planet gear with a thin rim, the shape of load distribution on the inner surface of the planet gear depends on the tooth load and the clearance. Therefore the nominal stress in the rim of the planet gear is unproportional to the tooth load in the case of the constant clearance and depends on the clearance in the case of the constant tooth load[2),3)]. Consequently it is no longer easy to obtain the nominal stress in the rim of the

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planet gear with a thin rim.

In this report, a theoretical equation of the nominal stress in the rim of the planet gear is deduced approximately by using an equivalent ring[4] and the theory of thin ring. Further by using the equation obtained, effects of the tooth load and the clearance on the stress in the rim are investigated.

Nomenclature

P_n : Tooth load
P_{ct} : Circumferential component of the tooth load
P_{cr} : Radial component of the tooth load
M_c : Moment acting in the rim due to the tooth load
h : Rim thickness of the planet gear
h_e : Thickness of the equivalent ring of the planet gear
r : Radius of curvature of the center line of the rim of planet gear
r_e : Radius of curvature of the center line of the equivalent ring
θ_i : Central angle showing the region where the rim is out of contact with the gear shaft
β : Central angle showing the position of tooth load
E : Longitudinal elastic modulus
I_e : Moment of inertia of cross section of the equivalent ring
Δr : Radial clearance between the rim and the gear shaft
b : Face width
η : Distance from the center line of equivalent ring to the intersection of line of action and the center line of loaded tooth
δ : Displacement of the rim
i : Angular displacement of the rim
σ : Stress

2. Nominal Bending Stress in the Rim of Planet Gear

2.1 Equivalent Model of Planet Gear with Thin Rim

As the form of the planet gear is complex, the planet gear which has the rim thickness h and the radius of curvature of the center line of the rim r is replaced by the equivalent ring which has the thickness h_e and the radius of curvature of the center line r_e in the analysis of the nominal stress in the rim as shown in Fig. 1. Further the tooth load acted on the planet gear P_n is resolved into the circumferential force P_{ct}, the radial force P_{cr} and the moment M_c ($= P_{ct}\eta$) acted on the center line of the equivalent ring as shown in Fig. 1. Where η denotes the distance from the intersection of the line of action and the center line of the loaded tooth to the center line of the equivalent ring. Moreover a curved beam model which central angle is 180° is used as shown in Fig. 2 because the tooth load sets are applied at the points which are almost symmetry each other to the axe 0–0' as shown in Fig. 1 and the following boundary conditions are used : The section A is clamped rigidly and the section D can move only radially as shown in Fig. 2. Further it is assumed that the contact region is in the region $\beta \leqq \psi \leqq \pi$, where the position of the tooth load is $\psi = \beta$.

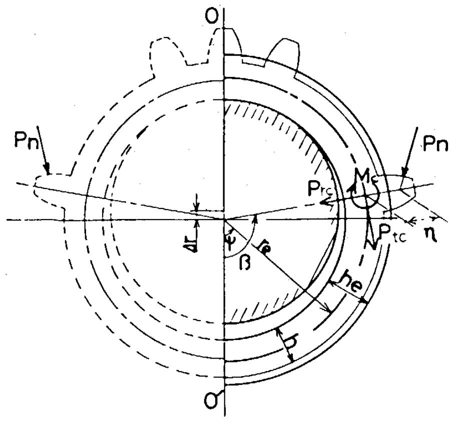

Fig. 1 Schema of the planet gear and the equivalent ring

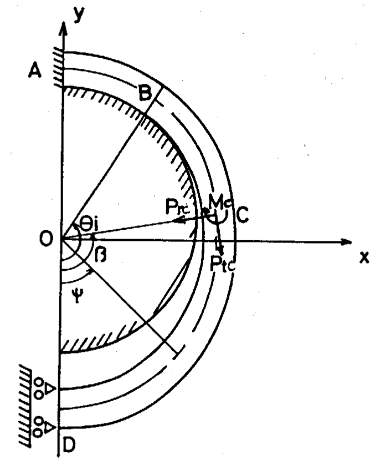

Fig. 2 Curved beam model used

2.2 Analytical Method

2.2.1 Outline of Analytical Method

Although the module, the number of teeth and the rim thickness are all constant, the amount of contact region depends on both the tooth load and the radial clearance in the case of the planet gear with a thin rim and the radial clearance. Further, the amount of contact region affects the nominal stress in the rim. Therefore in order to obtain the nominal stress in the rim, the amount of the contact region will be determined in the first place. In the analysis of the amount of the contact region, it is assumed that the rim is in contact with the gear shaft in the region $\theta_i \leqq \psi \leqq \pi$ as shown in Fig. 2 and then the analysis is done in two regions which are the region $\theta_i \leqq \psi \leqq \pi$ and that $0 \leqq \psi \leqq \theta_i$ where the rim is out of contact with the gear shaft. Further, after the bending moment at the assumed position $\psi = \theta_i$ in each region is analyzed and compared with each other, the position where the bending moment in the one region is equal to that in the other one is put as the true boundary position. Then using the true boundary position $\psi = \theta_i$, the bending moment and the circumferential force in the rim are obtained and also the nominal stress is obtained.

2.2.2 Bending Moment in Contact Region

The bending moment $M(\psi)$ in the contact region $\theta_i \leqq \psi \leqq \pi$ is constant as shown in the next expression

$$M(\psi) = EI_e \left\{ \frac{1}{r_e - \Delta r} - \frac{1}{r_e} \right\} \qquad \cdots\cdots (1)$$

where the bending moment which increases the curvature of the ring is put as plus. E and I_e are the longitudinal elastic modulus and the moment of inertia of the coss section of the equivalent ring respectively and Δr is the radial clearance between the rim and the gear shaft.

2.2.3 Bending Moment in Uncontact Region

If the rim is in contact with the gear shaft in the region $\theta_i \leqq \psi \leqq \pi$ as mentioned above, it may be reasonable to assume that the curved beam model is clamped at the section B on the gear shaft as shown in Fig. 3(b) in the analysis of stress and deflection in the uncontact region $0 \leqq \psi \leqq \theta_i$. When the model is clamped at the section B and free at the section D in Fig. 2, the section D may move geometrically to D' which is located i_o in the angular

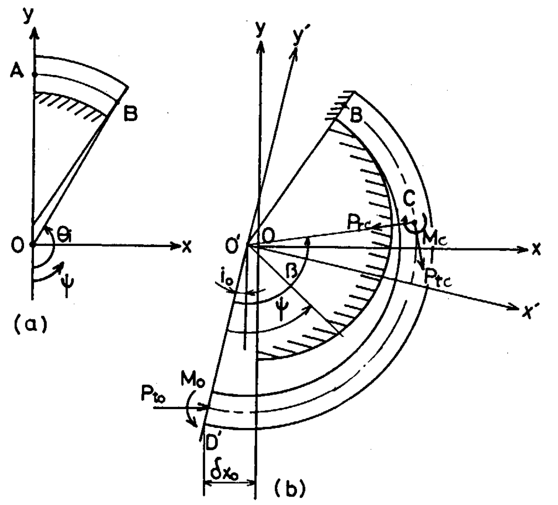

Fig. 3 Separation of the curved beam model in the analysis

displacement and δ_{xo} in the displacement from the y–axe as shown in Fig. 3 (b). The displacements i_o and δ_{xo} can be obtained geometrically in the next expressions.

$$i_o = \frac{\Delta r(\pi - \theta_i)}{r_e - \Delta r} \quad \cdots\cdots (2)$$

$$\delta_{xo} = (r_e + \Delta r) \sin \left\{\frac{\Delta r(\pi - \theta_i)}{r_e - \Delta r}\right\} \quad \cdots\cdots (3)$$

In case where the section D' is located i_o and δ_{xo} from y–axe, if the loads P_{ct} and P_{cr} and the moment M_c are applied at the section C where $\psi = \beta$ and further the load P_{to} and the moment M_o are applied at the section D' where $\psi = 0$, the bending moment $M(\psi)$ at the arbitrary position ψ is determined as follows.

$$\left.\begin{aligned} M(\psi) &= -M_o - P_{to} r_e \, (\mathrm{C0} - \mathrm{C5}) && (0 \leqq \psi \leqq \beta) \\ M(\psi) &= -M_o - P_{to} r_e \, (\mathrm{C0} - \mathrm{C5}) \\ &\quad + M_c + P_{ct} r_e \, (1 - \mathrm{C1}) - P_{cr} r_e e \mathrm{S1} && (\beta_i \leqq \psi \leqq \theta) \end{aligned}\right\} \cdots (4)$$

Symbols eg. CO, C1 in Eq. (4) are shown in Table 1.

Elastic strain energy U in the region $0 \leqq \psi \leqq \theta_i$ can be expressed in the next form by using the bending moment $M(\psi)$ shown in Eq. (4).

$$U = \frac{r_e}{2EI_e} \int_0^{\theta_i} M(\psi)^2 d\psi \quad \cdots\cdots (5)$$

Table 1 Symbols of trigonometrical functions

$S0 = \sin i_o$	$C0 = \cos i_o$
$S1 = \sin(\beta - \psi)$	$C1 = \cos(\beta - \psi)$
$S2 = \sin(\theta_i - \psi)$	$C2 = \cos(\theta_i - \psi)$
$S3 = \sin(\theta_i - \beta)$	$C3 = \cos(\theta_i - \beta)$
$S4 = \sin(\theta_i - i_o)$	$C4 = \cos(\theta_i - i_o)$
$S5 = \sin(i_o - \psi)$	$C5 = \cos(i_o - \psi)$
$S6 = \sin\theta_i$	$C6 = \cos\theta_i$
$S7 = \sin\beta$	$C7 = \cos\beta$

Both the circumferential displacement δ_{xo} and the angular displacement i_o at the section D must be zero in the planet gear. Therefore D' in Fig. 3 (b) must be moved i_o expressed in Eq. (2) and δ_{xo} expressed in Eq. (3) respectively by the forces P_{to}, P_{ct}, P_{cr} and the moments M_o, M_c. Using these condition, the relations among the strain energy U, the force and the moment P_{to}, M_o and the displacements i_o, δ_{xo} can be expressed as follows.

$$\delta_{xo} = \frac{\partial U}{\partial P_{to}},\quad i_o = \frac{\partial U}{\partial M_o} \quad \cdots\cdots (6)$$

From Eqs. (4), (5) and (6), P_{to} and M_o are obtained as follows.

$$\left.\begin{aligned} P_{to} &= \frac{EI_e}{r_e^2 J}\left\{-\theta_i K + r_e(\theta_i C0 - S4 - S0)L\right\} \\ M_o &= \frac{EI_e}{r_e^2 J}\left\{\frac{1}{r_e}(-\theta_i C0 + S4 + S0)K - \frac{\theta_i}{2}\right. \\ &\quad \left. + \frac{S4\cdot C4}{2} + \frac{S0\cdot C0}{2} + \theta_i C0^2 - 2S4\cdot C0 - 2S0\cdot C0)\ L\right\} \end{aligned}\right\} \quad \cdots\cdots (7)$$

Symbols J, K and L in Eq. (7) are as shown in the next equation.

$$\left.\begin{aligned} J &= \frac{\theta_i^2}{2} + \frac{\theta_i S4\cdot C4}{2} + \frac{\theta_i S0\cdot C0}{2} - (S0 + S4)^2 \\ K &= \delta_{xo} + \frac{r_e}{EI_e}(a_{k1}M_c r_e + a_{k2}P_{ct}r_e^2 + a_{k3}P_{cr}r_e^2) \\ L &= i_o + \frac{r_e}{EI_e}(a_{l1}M_c + a_{l2}P_{ct}r_e + a_{l3}P_{cr}r_e) \end{aligned}\right\} \quad \cdots\cdots (8)$$

Coefficients a_{ki}, a_{li} in Eq. (8) are shown in Table 2. The bending moment at the section B in the uncontact region shown in Fig. 3 (b) is obtained in the next form by substituting the force

Table 2 Coefficients in Eq. (8)

a_{k1}	$(C6-C7)\,S0+(\theta_i-\beta-S6+S7)\,C0$
a_{k2}	$\{C6-(3/4)C7+(1/2)(\theta_i-\beta)S7+(1/4)(S3\cdot S6-C3\cdot C6)\}\,S0+\{\theta_i-\beta-S3-S6+(3/4)S7+(1/2)(\theta_i-\beta)C7+(1/4)(S3\cdot C6-C3\cdot S6)\}\,C0$
a_{k3}	$\{-(1/4)S7-(1/2)(\theta_i-\beta)C7+(1/4)(S3\cdot C6+C3\cdot S6)\}\,S0+\{-C3+1+(1/2)(\theta_i-\beta)\,S7+(1/4)C7+(1/4)(C3\cdot C6-S3\cdot S6)\}\,C0$
a_{l1}	$\theta_i-\beta$
a_{l2}	$(\theta_i-\beta)-S3$
a_{l3}	$1-C3$

P_{to} and the moment M_o shown in Eq. (7) into Eq. (4).

$$M(\theta_i)=M_o+P_{to}r_e(C0-C5)+M_c-P_{cr}r_eS1+P_{ct}r_e(1-C1) \qquad \cdots\cdots\cdots (9)$$

The section B is the boundary between the contact region and the uncontact one. Therefore if θ_i is reasonable, $M(\theta_i)$ in Eq. (9) must be equal to $M(\psi)$ obtained by Eq. (1). Consequently using the angle $\psi=\theta_i$ where the condition $M(\theta_i)=M(\psi)$ is satisfied, the force P_{to} and the moment M_o are determined by Eq. (7) and further the bending moment in the uncontact region $0\leqq\psi\leqq\theta_i$ is obtained by Eq. (4).

2.2.4 Circumferential Force

The circumferential force in the equivalent ring $N(\psi)$ is obtained by the next expression in the uncontact region $0\leqq\psi\leqq\theta_i$.

$$\left.\begin{aligned} N(\psi)&=P_{to}C5 && (0\leqq\psi\leqq\beta)\\ N(\psi)&=P_{to}C5-P_{cr}S1+P_{ct}C1 && (\beta\leqq\psi\leqq\theta_i)\end{aligned}\right\} \qquad \cdots\cdots\cdots (10)$$

where the sign of $N(\psi)$ is put as plus if $N(\psi)$ is tensile force. Whereas $N(\psi)$ in the contact region $\theta_i\leqq\psi\leqq\pi$ is assumed to be constant because of thin rim, which is obtained by substituting $\psi=\theta_i$ into the second expression of Eq. (10).

2.3 Nominal Stress in the Rim

Stress in the rim may be mainly composed of the bending stress and the circumferential stress neglecting the shear stress which is relatively small in the thin rim. The nominal bending stress σ_M on the tooth fillet is expressed in the next form in term of the bending moment $M(\psi)$ described in 2. 2. 2 and 2. 2. 3.

$$\sigma_M = \frac{6M(\psi)}{bh^2} \quad \cdots\cdots (11)$$

where b denotes the face width.

Whereas the circumferential stress σ_N is expressed in the next form in term of the circumferential force described in 2. 2. 4 :

$$\sigma_N = \frac{N(\psi)}{bh} \quad \cdots\cdots (12)$$

3. Results and Discussions

3.1 Comparison of Theoretical Value with Experimental One

Stress on the inner surface of the rim of the external gear is little dependent on the stress concentration at the tooth fillet. Therefore the stress on the inner surface of the rim of the planet gear ($-\sigma_M + \sigma_N$) is calculated by Eqs. (10) and (12) and shown in Fig.4 with the

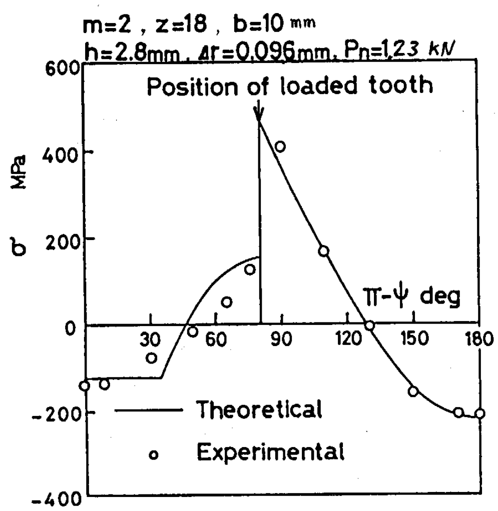

Fig. 4 Stress distribution on the inner surface of the rim

stress obtained by the photoelastic method in the case of the module $m=2$, the number of teeth $z=18$, the face width $b=10$mm, the rim thickness $h=2.8$mm, the radial clearance $\Delta r=0.096$mm and the tooth load $P_n=1.23$kN. In Fig. 4, the calculated result fairly coincides with the experimental one. Therefore the nominal stress in the rim of the planet gear can be obtained by the analytical method described above.

3.2 Effects of Tooth Load and Radial Clearance on Nominal Stress in the Rim

It has been seen by the photoelastic investigation that the fillet stress in the planet gear with both the thin rim and the clearance between the rim and the gear shaft depends on the clearance in the case of constant tooth load and is unproportional to the tooth load in the case of constant clearance[2]. These effects may be owing to the nominal stress in the rim. Fig. 5 shows the relation between the stress on the inner surface of the planet gear ($-\sigma_M + \sigma_N$) obtained by Eqs. (11) and (12) and the clearance under the condition of constant ratio of the clearance to the tooth load $\Delta r/P_n$. Also the central angles of the contact region ($\pi - \theta_i$) are shown in Fig. 5. It is seen from Fig. 5 that the stresses on the rim are proportional to either the tooth load or the clearance like as the fillet stress obtained by the experiment[2] under

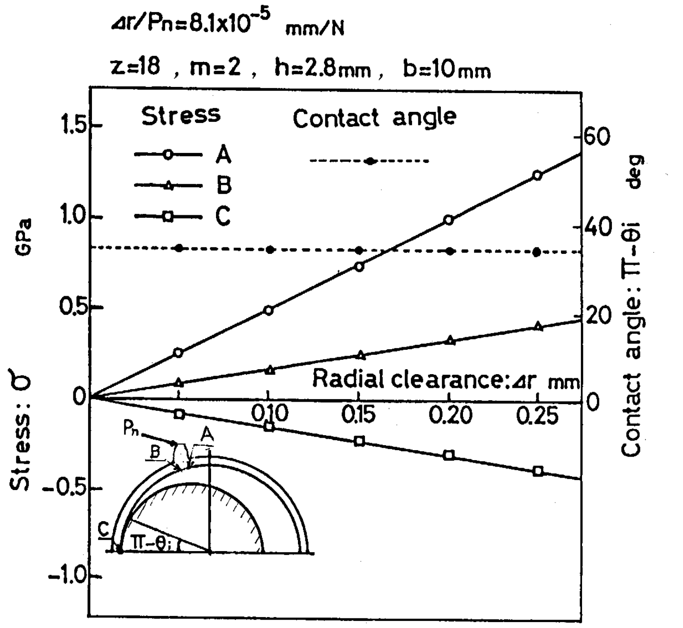

Fig. 5 Effect of the radial clearance on the stress in the rim in the case of the constant ratio of the radial clearance to the tooth load

condition of the constant ratio of $\Delta r/P_n$. Moreover it is clear that the contact region is independent on both the tooth load and the clearance under the condition of the constant ratio of $\Delta r/P_n$.

4. Conclusion

The planet gear with both the thin rim and the clearance between the rim and the gear shaft are replaced by the equivalent ring. Then considering the effects of both the tooth load and the clearance on the region of the contact between the rim and the gear shaft, the nominal stress in the rim of the planet gear is analyzed theoretically. Further the effects of the tooth load and the clearance on the nominal stress are investigated as follows :

(1) Stress on the inner surface of the planet gear obtained theoretically coincides with that obtained by the photoelastic method.

(2) If the ratio of the tooth load to the clearance between the rim and the gear shaft is constant, the nominal stress in the rim of the planet gear with the clearance is proportional to either the tooth load or the clearance and further the amount of contact region is constant.

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