Object Manipulation under Hybrid Active/Passive Closure

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Abstract— In this paper, we discuss the manipulation of an object under hybrid active/passive closure. We show the orthogonality between the directions of active and passive force closures for general grasping systems. Based on the orthogonality, we decompose the dynamics of grasping system into the "active part" and the "passive part". By using the decomposition, we show that the grasped object can be manipulated only by considering the dynamics of the active part. We also consider how to determine the desired internal forces in order to satisfy frictional constraints during the manipulation. In order to verify the validity of our approach, some simulation results are shown.

Index Terms— Active/Passive Force Closure, Manipulation, Orthogonality, Grasping

I. INTRODUCTION

The force closure has been one of the important properties of grasping [1]. However, the force closure had been interpreted by two ways [2]. One is that " any arbitrary force and moment can be applied to a grasped object by fingers. " and the other is that " any arbitrary external force and moment exerted on a grasped object can be balanced without changing the pre-loaded joint torques and the motion of the object can be completely constrained."

The former concept corresponds to that fingers (limbs) can move a grasped object in arbitrary directions. On the other hand, the latter concept corresponds to the power grasps [3] where fingers can grasp object stably without changing the pre-loaded joint torques. In order to remove the ambiguity of the definition, Yoshikawa [4] named the former concept active force closure, and the latter concept passive force closure.

Here, we can encounter many cases where a grasping system has both the active and the passive closure properties. Yoshikawa [4] called such grasps as hybrid active/passive-closure grasps. For the hybrid active/passive closure properties included in many grasping systems, the key issue is the orthogonality between the "active" and the "passive" parts. While we introduced the orthogonality between them [5], the orthogonality has been proven for a limited class of grasping systems. Also, the control algorithm for manipulating an object has never been considered.

In this paper, we show the orthogonality between the active and the passive parts for general grasping systems. Also, we propose a control algorithm for manipulation of an object considering the hybrid active/passive closure

properties for general grasping systems. While our algorithm can be applicable for general grasping systems, it has the following merit; When controlling the motion of the grasped object, our algorithm does not need to consider the whole dynamics of the grasping systems. This is due to the orthogonal property between the active and the passive parts of the grasping system.

Previously, hybrid position/force control has been proposed for several robot manipulators [6]. Our proposed algorithm can include the conventional hybrid control algorithm. However, the extension is not straightforward. Different from the robot manipulators, a grasping system has uncontrollable parts such as the passive part. Our proposed control algorithm can deal with such uncontrollable parts. Off course, the frictional constraints, which are not considered in the conventional hybrid control, also can be dealt with.

This paper is organized as follows. At first, the target system is shown and the directions of active and passive force closures are defined. Then, we show the directions of active force closure are orthogonal to those of passive force closure. Using the orthogonality, we derive a control algorithm. We also discuss how to determine the desired internal forces for the manipulation. Lastly, we show some simulation results to show the validity of our approach.

A. Related Works

Firstly Trinkle [2] gave force closure two interpretations as described above. Yoshikawa [4] named the two concepts active and passive force closures. Previously, the active and the passive force closures have been studied separately.

As for active force closure, Li et al. [7], Cole et al. [8], and Yokokohji et al. [9] presented control algorithms for the case of point contact, rolling contact, and soft-finger contact, respectively, for the manipulation of an object grasped by fingertips. Harada et al. analyzed the active force closure [10] and presented a control algorithm [11] for manipulating multiple objects. There are some researches focusing on active force closure in the general grasping systems. Trinkle et al. [12] discussed a grasp planning for manipulating an enveloped object with sliding contacts in 2 dimensional space. Bicchi et al. [13] analyzed the manipulability of the general grasping systems. Harada et al. [14] presented a sufficient condition for the manipulation of Envelope Family and realize the manipulation

with constant joint driving torques. Park et al. [15] derived contact forces and accelerations consistent with dynamics and friction law for a given torque-wrench pair.

Note that there exist two ways as which we can regard active force closure. One is that fingers (limbs) can move a grasped object in arbitrary directions as described above. The other is that active force closure is standard notation of force closure and is necessary but not sufficient for passive force closure [16]. We follow the precedent active force closure by Yoshikawa [4] by adopting the following definition.

Active force closure : A grasp is said to be of active force closure if any resultant force and moment can be applied to the object by fingers and the maintenance of the object's equilibrium requires the application of the resultant force and moment applied by the fingers.

On the other hand, the passive force closure (power grasp) have been researched with respect to the analysis of robustness, the analysis of indeterminate forces, the formulation of contact force distribution, the optimization of pre-loaded joint torques, and so on [16]-[24].

There are many cases where a grasping system has both active and passive force-closure properties simultaneously. Only recently, the authors have begun some researches focusing on the both properties [5], [25]. However, the obtained results are still limited. In this paper, we show the orthogonality between the active and the passive parts for general grasping systems, and present a control algorithm for the manipulation, based on the orthogonality.

II. NOMENCLATURE

The following nomenclatures are used in this paper.

N Number of fingers.

M_i Number of joints of the ith finger ($i = 1, 2, \cdots, N$).

L_i Number of contact points on the ith finger.

M Number of total joints ($= \Sigma_{i=1}^N M_i$).

L Number of total contact points ($= \Sigma_{i=1}^N L_i$).

D 3/6 in 2/3 dimensional space.

d 2/3 in 2/3 dimensional space.

Σ_R Reference coordinate frame.

Σ_O Object coordinate frame fixed at the object.

Σ_G Coordinate frame fixed at the center of gravity of the object.

C_{ij} The jth contact point of the ith finger ($j = 1, 2, \cdots, L_i$).

$\Sigma_{C_{ij}}$ Coordinate frame fixed at C_{ij}.

$\Sigma_{F_{ij}}$ Coordinate frame fixed at the link of the ith finger, on which there exists C_{ij}.

$q_i \in \mathcal{R}^{M_i}$ Joint angle vector of the ith finger.

$p_I \in \mathcal{R}^d$ Position of the origin of Σ_I ($I = O, G, C_{ij}, F_{ij}$).

$\omega_I \in \mathcal{R}^{D-d}$ Rotational velocity vector of Σ_I.

$\nu_{C_{ij}} \in \mathcal{R}^d$ Contact point velocity at C_{ij} where the component of contact point motion due to rolling is excluded ($\nu_{C_{ij}} = \dot{p}_{C_{ij}}$ where rolling doesn't occur).

$v_I \in \mathcal{R}^D$ $\left(\dot{p}_I^T \ \omega_I^T \right)^T$.
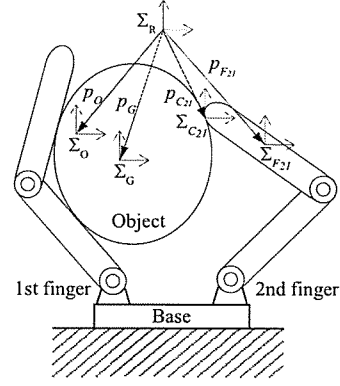

Fig. 1. Target System ($N = 2$)

$w_I \in \mathcal{R}^D$ Resultant force and moment applied to the object at Σ_I ($I = G, O$).

$f_{ij} \in \mathcal{R}^d$ Contact force vector at C_{ij}.

$f \in \mathcal{R}^{Ld}$ $\left(f_{11}^T \ f_{12}^T \ \cdots \ f_{NL_N}^T \right)^T$.

$\tau_c \in \mathcal{R}^M$ Joint torque vector equivalent to f.

$n_{f_{ij}}$ Normal force component of f_{ij}.

$t_{f_{ij},k}$ Tangential force components of f_{ij} ($k = 1, 2$).

μ_{ij} Frictional coefficient at C_{ij}.

Note that since there exist some cases where it is convenient if the origin of Σ_O doesn't coincide with the center of gravity of the object, we define not only Σ_O but also Σ_G.

III. TARGET SYSTEM AND DEFINITION

A. Target System

The target system is shown in Fig.1. In this paper, we consider the case where an arbitrary shaped rigid object is grasped by N fingers of a robotic hand. We make the following assumptions: 1) Each finger makes a frictional point contact with the object, and sliding doesn't occur at each contact point. 2) There exists at most one contact point on each link of the fingers. 3) The number of contact points does not change and the remove of contact points does not occur.

B. Definition

When manipulating an object under hybrid active/passive closure, the directions of object motion is limited to the directions satisfying active force closure. To obtain a controller for manipulating an object, we decompose the dynamics of grasping system into the "active part" and the "passive part". Here, we do not control the object motion within the directions satisfying passive force closure since wrench applied on the object is counteracted without any additional joint torque input. For such purpose, we introduce the following two spaces within the generalized velocity space and the generalized force space (wrench space) of the object.

Space of Active Force Closure (SAFC); Let us consider the direction in which a grasped object can move

within a constraint applied by fingers. If the fingers can do positive work on the object in that direction, we call the direction as direction of active force closure (DAFC). Also, we call the space, spanned by a set of the all DAFC's, as space of active force closure (SAFC).

Space of Passive Force Closure (SPFC); We call the direction, in which external force or moment can be counteracted without changing the joint torques, as direction of passive force closure (DPFC). Also, we call the space, spanned by a set of the all DPFC's, as space of passive force closure (SPFC).

IV. BASIC FORMULATION OF THE SYSTEM

We discuss 3 dimensional case, but the formulation can be easily extended to 2 dimensional case.

A. Kinematics of the system

The relation between v_O and v_G is given by

$$v_G = D_{GO} v_O \qquad (1)$$

where

$$D_{GO} = \begin{pmatrix} I & -[(p_G - p_O) \times] \\ O & I \end{pmatrix} \in \mathcal{R}^{D \times D}$$

where I represents an identity matrix, O represents a zero matrix, $[a \times]$ represents a skew symmetric matrix equivalent to the cross product operation ($[a \times]b = a \times b$).

The relation between $v_{F_{ij}}$ and $\dot{q}_i$, and the relation among $\nu_{C_{ij}}$, $v_{F_{ij}}$ and v_O, respectively, are given as follows;

$$v_{F_{ij}} = J_{F_{ij}} \dot{q}_i, \quad G^T_{O_{ij}} v_O = \nu_{C_{ij}} = D^T_{F_{ij}} v_{F_{ij}} \qquad (2)$$

where $J_{F_{ij}} \in \mathcal{R}^{D \times M_i}$ denotes the Jacobian matrix and

$$G_{O_{ij}} = \begin{pmatrix} I \\ [(p_{C_{ij}} - p_O) \times] \end{pmatrix} \in \mathcal{R}^{D \times d},$$

$$D_{F_{ij}} = \begin{pmatrix} I \\ [(p_{C_{ij}} - p_{F_{ij}}) \times] \end{pmatrix} \in \mathcal{R}^{D \times d}.$$

By using the following vectors and matrices,

$$\dot{q} = \begin{pmatrix} \dot{q}^T_1 & \dot{q}^T_2 & \cdots & \dot{q}^T_N \end{pmatrix}^T \in \mathcal{R}^M,$$

$$J = \text{diag} \left(\begin{pmatrix} D^T_{F_{11}} J_{F_{11}} \\ \vdots \\ D^T_{F_{1L_1}} J_{F_{1L_1}} \end{pmatrix} \begin{pmatrix} D^T_{F_{21}} J_{F_{21}} \\ \vdots \\ D^T_{F_{2L_2}} J_{F_{2L_2}} \end{pmatrix} \right.$$
$$\left. \cdots \begin{pmatrix} D^T_{F_{N1}} J_{F_{N1}} \\ \vdots \\ D^T_{F_{NL_N}} J_{F_{NL_N}} \end{pmatrix} \right) \in \mathcal{R}^{Ld \times M},$$

$$G_O = \begin{pmatrix} G_{O_{11}} & G_{O_{12}} & \cdots & G_{O_{NL_N}} \end{pmatrix} \in \mathcal{R}^{D \times Ld},$$

$$A_O = \begin{pmatrix} J & -G^T_O \end{pmatrix} \in \mathcal{R}^{Ld \times (M+D)}$$

where diag denotes a block diagonal matrix, from (2) we obtain

$$A_O \begin{pmatrix} \dot{q}^T & v^T_O \end{pmatrix}^T = o, \qquad (3)$$

where o denotes a zero vector. By solving (3), we obtain

$$\begin{pmatrix} \dot{q} \\ v_O \end{pmatrix} = E^T_P \dot{\zeta} = \begin{pmatrix} E^T_{P1} \\ E^T_{P2} \end{pmatrix} \dot{\zeta} \qquad (4)$$

where $E_P \in \mathcal{R}^{a \times (M+D)}$ denotes an orthogonal matrix whose rows form bases of the null space of A_O, $\zeta \in \mathcal{R}^a$ is an arbitrary vector expressing the magnitude of each column of E^T_P. Note that $\dot{\zeta}$ represent the object motion within the constraint applied by the fingers, and can be selected arbitrary.

B. Statics of the system

From (1) and the principle of virtual work, the relation between w_O and w_G is given by

$$w_O = D^T_{GO} w_G. \qquad (5)$$

From (3) and the principle of virtual work, the relation among f, w_O and τ_c is given by

$$\begin{pmatrix} \tau^T_c & -w^T_O \end{pmatrix}^T = A^T_O f = \begin{pmatrix} J & -G^T_O \end{pmatrix}^T f. \qquad (6)$$

From (6), the following relation is obtained;

$$f = (J^T)^+ \tau_c + (I - (J^T)^+ J^T) \check{k}_1, \qquad (7)$$

where $(J^T)^+$ denotes the pseudo-inverse matrix of J^T and $\check{k}_1 \in \mathcal{R}^{Ld}$ denotes an arbitrary vector. Note that the force of the second term in the right side of (7) expresses an internal force which makes no influence on the joint torques.

By substituting (7) into (6), we get

$$\begin{pmatrix} \tau_c \\ -w_O \end{pmatrix} = \begin{pmatrix} I \\ -G_O(J^T)^+ \end{pmatrix} \tau_c + \begin{pmatrix} O \\ -G_O(I - (J^T)^+ J^T) \end{pmatrix} \check{k}_1$$
$$\triangleq \begin{pmatrix} I \\ -G_{OJ} \end{pmatrix} \tau_c + \begin{pmatrix} O \\ -\Xi \end{pmatrix} k_1, \qquad (8)$$

where $G_{OJ} = G_O(J^T)^+$, $\Xi \in \mathcal{R}^{D \times p}$ is an orthogonal matrix whose columns form bases of the $G_O(I - (J^T)^+ J^T)$, and $k_1 \in \mathcal{R}^p$ is an arbitrary vector expressing the magnitude of each column of Ξ. Note that the second term in the right side of (8) can express a generable resultant force without changing the joint torques.

C. Frictional constraints

The frictional constraint at C_{ij} ($i = 1, 2, \cdots, N$, $j = 1, 2, \cdots, L_i$) can be represented by

$$\mathcal{F}_{fij} = \{f_{ij} | \sqrt{t^2_{fij,1} + t^2_{fij,2}} \le \mu_{ij} n_{fij}, \ n_{fij} \ge 0\}. \qquad (9)$$

Aggregating (9) for all contact points, we obtain

$$\mathcal{F}_f = \{f | f_{ij} \in \mathcal{F}_{fij}, \ \forall f_{ij}\}. \qquad (10)$$

D. Dynamics of the object and the fingers

The equation of motion of the object and the fingers, respectively, can be represented as follows;

$$M_r \dot{v}_G + h_r = w_G = D_{GO}^{-T} w_O, \quad (11)$$

$$M_q \ddot{q} + h_q + \tau_c = \tau \quad (12)$$

where τ denotes the joint driving torques, M_r and M_q are the inertia tensors of the object and the fingers, respectively, h_r and h_q are the terms representing centrifugal, Coriolis and gravitational forces of the object and the fingers, respectively.

From (1) and (5), (11) can be rewritten by

$$M_{Or} \dot{v}_O + h_{Or} = w_O = G_O f, \quad (13)$$

where

$$
\begin{aligned}
M_{Or} &= D_{GO}^T M_r D_{GO}, \\
h_{Or} &= D_{GO}^T M_r \dot{D}_{GO} v_O + D_{GO}^T h_r.
\end{aligned}
$$

V. ORTHOGONALITY BETWEEN SAFC AND SPFC

In this section, we show the orthogonality between SAFC and SPFC [5].

Let us consider the case where the object is stably grasped with τ_c and is in steady state. Namely, from (8) and (10), τ_c hold

$$\begin{pmatrix} \tau_c \\ -w_{O_{st}} \end{pmatrix} = A_O^T f_{st} = \begin{pmatrix} I \\ -G_{OJ} \end{pmatrix} \tau_c + \begin{pmatrix} O \\ -\Xi \end{pmatrix} k_{1_{st}}, \quad f_{st} \in \mathcal{F}_f \quad (14)$$

where $w_{O_{st}}$, f_{st} and $k_{1_{st}}$, respectively, are w_O, f and k_1 in the case where the system is in the steady state. SAFC and SPFC are considered in this case.

A. SAFC

At first, we consider a canonical/reduced form of E_{P2}. E_P given in (4) can be written as follows (refer to [13]);

$$\begin{pmatrix} E_{P1}^T \\ E_{P2}^T \end{pmatrix} = \begin{pmatrix} E_{P11}^T & E_{P12}^T & O \\ O & E_{P22}^T & E_{P23}^T \end{pmatrix}. \quad (15)$$

Here $\mathrm{Im}(E_{P11}^T)$ ($= \ker(J)$) denotes the redundancy subspace of joint velocities that do not affect the object velocities, and $\mathrm{Im}(E_{P23}^T)$ ($= \ker(G_O^T)$) denotes the indeterminacy subspace of object velocities. Then, the directions indicated by the columns of $(E_{P22}^T \ E_{P23}^T)$ is the directions in which the object can move within the constraint applied by the fingers. Here, we consider the following transformation;

$$
\begin{aligned}
\begin{pmatrix} E_{P1}^T \\ E_{P2}^T \end{pmatrix} \Psi &= \begin{pmatrix} E_{P11}^T & E_{P12}^T & O \\ O & E_{P22}^T & E_{P23}^T \end{pmatrix} \begin{pmatrix} I & O & O \\ O & \Psi_2 & O \\ O & O & \Psi_3 \end{pmatrix} \\
&\triangleq \begin{pmatrix} E_{P11}^T & \tilde{E}_{P1}^T \\ O & \tilde{E}_{P2}^T \end{pmatrix},
\end{aligned} \quad (16)
$$

$$\zeta = \Psi \begin{pmatrix} \dot{\bar{\zeta}}^T & \dot{\tilde{\zeta}}^T \end{pmatrix}^T, \quad (17)$$

where $\Psi \in \mathcal{R}^{a \times a}$ is a nonsingular matrix for the transformation which makes both $E_{P22}^T \Psi_2$ and $E_{P23}^T \Psi_3$ be orthogonal matrices. Note that since ζ is arbitrary, $\dot{\bar{\zeta}} \in \mathcal{R}^{a-\bar{a}}$ and $\dot{\tilde{\zeta}} \in \mathcal{R}^{\bar{a}}$ are also arbitrarily and express the magnitude of each column of E_{P11}^T, and that of $E_{P22}^T \Psi_2$ ($E_{P12}^T \Psi_2$) and $E_{P23}^T \Psi_3$, respectively. Note also that $\tilde{E}_{P1}^T \in \mathcal{R}^{M \times \bar{a}}$ and $\tilde{E}_{P2}^T \in \mathcal{R}^{D \times \bar{a}}$.

From the transformation, we obtain

$$\begin{pmatrix} E_{P1}^T \\ E_{P2}^T \end{pmatrix} \zeta = \begin{pmatrix} E_{P11}^T & \tilde{E}_{P1}^T \\ O & \tilde{E}_{P2}^T \end{pmatrix} \begin{pmatrix} \dot{\bar{\zeta}} \\ \dot{\tilde{\zeta}} \end{pmatrix}. \quad (18)$$

Note that $\mathrm{Im}(\tilde{E}_{P1}^T) \in \mathrm{Im}(J^T)$ because $\mathrm{Im}(E_{P1}^T) \cap \ker(J) = \mathrm{Im}(E_{P11}^T)$.

Then, the allowable motions of the object are given by

$$\mathcal{A} = \{\Delta r | \Delta r = \tilde{E}_{P2}^T \Delta \tilde{\zeta}\} \quad (19)$$

where Δr denotes the displacements of the object.

From (8), the work done by τ_c is given by

$$\text{Work} = \Delta r^T w_O = \Delta r^T G_{OJ} \tau_c + \Delta r^T \Xi k_1. \quad (20)$$

Note that Ξk_1 is resultant force and moment which work to counteract external force and moment in the direction contained in $\mathrm{Im}(\Xi)$, only when the external force and moment are exerted on the object. Then, the applied force and moment by τ_c in the direction contained in $(\mathrm{Im}(G_{OJ}) \cap \mathrm{Im}(\Xi))$ will be counteracted. Then, defining E so that $(E \ \Xi) \in \mathcal{R}^{D \times D}$ is an orthogonal matrix, from (14) and (19), DAFC can be described as follows;

$$
\begin{aligned}
\text{DAFC} = \{ \Delta r | &\text{Work} = \Delta r^T E E^T G_{OJ} \tau_c > 0, \ \Delta r \in \mathcal{A}, \\
&(I - EE^T) G_{OJ} \tau_c + G_O (I - (J^T)^+ J^T) \tilde{k}_1 = o, \\
&(J^T)^+ \tau_c + (I - (J^T)^+ J^T) \tilde{k}_1 + f_{st} \in \mathcal{F}_f \}. \quad (21)
\end{aligned}
$$

When any $(J^T)^+ \tau_c$, whose magnitude is finite, can be applied to the object for the appropriately assigned f_{st}, (21) becomes

$$\text{DAFC} = \{\Delta r | \Delta r = \tilde{E}_{P2}^T \tilde{E}_{P2} E E^T G_{OJ} \tau_c\}. \quad (22)$$

B. SPFC

SPFC is the space spanned by the generable resultant forces and moments without changing τ_c at the steady state. From (14), such resultant forces and moments, $w_{O_{ex}}$ hold

$$
\begin{aligned}
\begin{pmatrix} \tau_c^T & -w_{O_{ex}}^T & -w_{O_{st}}^T \end{pmatrix}^T &= A_O^T (f_{st} + f_{ex}) = \\
\begin{pmatrix} I & -G_{OJ} \end{pmatrix}^T \tau_c &+ \begin{pmatrix} O & -\Xi^T \end{pmatrix}^T (k_{1_{st}} + k_{1_{ex}}) \quad (23)
\end{aligned}
$$

where f_{ex} and $k_{1_{ex}}$, respectively, are f and k_1, which correspond to $w_{O_{ex}}$. Note that

$$\begin{pmatrix} o & -w_{O_{ex}}^T \end{pmatrix}^T = A_O^T f_{ex} = \begin{pmatrix} O & -\Xi^T \end{pmatrix}^T k_{1_{ex}}, \quad (24)$$

$$f_{ex} = (I - (J^T)^+ J^T) \tilde{k}_{1_{ex}}. \quad (25)$$

If $f_{st} + f_{ex} \in \mathcal{F}_f$, $\Xi k_{1_{ex}}$ becomes DPFC. Then, SPFC is given by

$$
\begin{aligned}
\text{SPFC} = \{ w | w = \Xi k_{1_{ex}}, f_{ex} = (I - (J^T)^+ J^T) \tilde{k}_{1_{ex}}, \\
f_{st} + f_{ex} \in \mathcal{F}_f \}. \quad (26)
\end{aligned}
$$

C. Orthogonality

In order to show the orthogonality between SAFC and SPFC, we have only to show that every possible DPFC is orthogonal to every possible DAFC. Then, let us consider the case where every column of $\tilde{E}_{P2}^T$ in (22) (or (21)) corresponds to DAFC and every column of Ξ in (26) corresponds to DPFC.

From the definition of E_P (see (4)), $A_O E_P^T = O$. Then, we obtain

$$E_P A_O^T f = o, \quad \forall f.$$

Then, with respect to f_{ex} in (25), we obtain

$$E_P A_O^T f_{ex} = o, \quad E_P A_O^T (I - (J^T)^+ J^T) k_{1_{ex}} = o.$$

Then, from (24), we obtain

$$E_P \begin{pmatrix} O & -\Xi^T \end{pmatrix}^T k_{1_{ex}} = o.$$

Since $k_{1_{ex}}$ is an arbitrary vector, we obtain

$$E_P \begin{pmatrix} O & -\Xi^T \end{pmatrix}^T = O, \quad E_{P2}\Xi = O,$$
$$\tilde{E}_{P2}\Xi = O. \tag{27}$$

(27) represents the orthogonality between SAFC (DAFC) and SPFC (DPFC).

Remark: Let us consider the generalized force space of object, spanned by the all columns of Ξ and $\tilde{E}_{P2}^T$. Using Ξ, the generalized force space of object can be represented by

$$\{w_o | w_o \in \mathcal{R}^D\} = \text{Im}(\Xi) \oplus \ker(\Xi^T)$$
$$= \text{Im}(\Xi) \oplus \ker((I - (J^T)^+ J^T) G_O^T).$$

$\ker((I - (J^T)^+ J^T) G_O^T)$ is given by

$$\ker((I - (J^T)^+ J^T) G_O^T)$$
$$= \ker(G_O^T) \cup \{w_o | G_O^T w_o \in \text{Im}(J)\}$$
$$= \{w_o | (x^T \ w_o^T)^T \in \ker(A_O), x \in \mathcal{R}^M\}$$
$$= \{w_o | w_o = \tilde{E}_{P2}^T x, x \in \mathcal{R}^{\tilde{a}}\}. \tag{28}$$

Then, the generalized force space of object can be spanned by the all columns of Ξ and $\tilde{E}_{P2}^T$, namely rank $(\Xi \ \tilde{E}_{P2}^T) = D$.

Using this orthogonality, (21) and (22), respectively, are rewritten by

$$\text{DAFC} = \{\Delta r | \text{Work} = \Delta r^T G_{OJ} \tau_c > 0, \ \Delta r \in \mathcal{A},$$
$$(I - \tilde{E}_{P2}^T \tilde{E}_{P2}) G_{OJ} \tau_c + G_O (I - (J^T)^+ J^T) k_1 = o,$$
$$(J^T)^+ \tau_c + (I - (J^T)^+ J^T) k_1 + f_{st} \in \mathcal{F}_f\}, \tag{21'}$$

$$\text{DAFC} = \{\Delta r | \Delta r = \tilde{E}_{P2}^T \tilde{E}_{P2} G_{OJ} \tau_c\}. \tag{22'}$$

VI. CONTROL LAW

In this section, a control law is derived. In order to manipulate the object under the hybrid active/passive closure, we assume that the space spanned by the all columns of $\tilde{E}_{P2}^T$ is SAFC and that the space spanned by the all columns of Ξ is SPFC. Note that these assumptions correspond to assume a force-closure grasp, which is needed to grasp and manipulate the object stably.

A. Decomposition of object dynamics into the Parts corresponding to SAFC and SPFC

We decompose the object dynamics into the part corresponding to SAFC and the part corresponding to SPFC. By using the decomposition, we have only to control the part corresponding to SAFC because any applied force and moment in the direction contained in SPFC are counteracted without any additional joint torque input and then control is not needed (if the frictional constraints are satisfied).

From (4), (8), (13) and (18), the dynamics of the object is rewritten by

$$M_{Or} \tilde{E}_{P2}^T \ddot{\zeta} + \tilde{h}_{Or} = w_o = G_{OJ} \tau_c + \Xi k_1, \tag{29}$$
$$\tilde{h}_{Or} = h_{Or} + M_{Or} \dot{\tilde{E}}_{P2}^T \dot{\zeta}.$$

By multiplying $\tilde{E}_{P2}$ to the both side of (29) from the left side and using (27) (the orthogonality between SAFC (DAFC) and SPFC (DPFC)), we obtain

$$\tilde{E}_{P2} M_{Or} \tilde{E}_{P2}^T \ddot{\zeta} + \tilde{E}_{P2} \tilde{h}_{Or} = \tilde{E}_{P2} G_{OJ} \tau_c. \tag{30}$$

This equation shows the component of object dynamics, which corresponds to SAFC. From the first assumption at the beginning of this section and (21') or (22'), $\tilde{E}_{P2} G_{OJ}$ has full row rank. Since M_{Or} is nonsingular and $\tilde{E}_{P2}$ has full row rank, $\tilde{E}_{P2} M_{Or} \tilde{E}_{P2}^T$ is also nonsingular. Hence, an arbitrary acceleration in the direction contained in SAFC can be generated by the joint torques.

From (30), we obtain

$$\tau_c = (\tilde{E}_{P2} G_{OJ})^+ (\tilde{E}_{P2} M_{Or} \tilde{E}_{P2}^T \ddot{\zeta} + \tilde{E}_{P2} \tilde{h}_{Or}) + \Phi k_2. \tag{31}$$

where $\Phi \in \mathcal{R}^{M \times (M - \tilde{a})}$ is a full column rank matrix whose columns form bases of the null space of $\tilde{E}_{P2} G_{OJ}$, and $k_2 \in \mathcal{R}^{(M - \tilde{a})}$ is an arbitrary vector expressing the magnitude of each column of Φ.

On the other hand, by multiplying Ξ^T to the both side of (29) from the left side, the part corresponding to SPFC is given by

$$k_1 = \Xi^T M_{Or} \tilde{E}_{P2}^T \ddot{\zeta} + \Xi^T \tilde{h}_{Or} - \Xi^T G_{OJ} \tau_c. \tag{32}$$

Note that substituting τ_c given by (31) and k_1 given by (32) into the right side of (29) and using the relation; $(\Xi \ \tilde{E}_{P2}^T)(\Xi \ \tilde{E}_{P2}^T)^T = I$, we obtain $o = o$.

B. Internal force

From (8), the following relation is obtained;

$$\tau_c = G_{OJ}^+ (w_o - \Xi k_1) + \Lambda \hat{f} \tag{33}$$

where $\Lambda \in \mathcal{R}^{M \times b}$ is a full column rank matrix whose columns consist of the bases contained in the space $(\text{Im}(J^T) \cap \ker(G_{OJ}))$, and $\hat{f} \in \mathcal{R}^b$ is an arbitrary vector expressing the magnitude of each column of Λ. Note that $(J^T)^+ \Lambda \hat{f}$ is the internal forces which can be controlled by the joint torques, τ_c $(G_O(J^T)^+ \Lambda \hat{f} = o)$. Note also that since τ_c is contained in $\text{Im}(J^T)$ from (6), Λ must be contained in $\text{Im}(J^T)$.

C. Dynamics of the total system

From (4), (7), (12) and (18), the dynamics of the fingers is rewritten by;

$$\tau = M_q(\tilde{E}_{P1}^T\ddot{\zeta} + E_{P11}^T\ddot{\zeta}) + \tilde{h}_q + \tau_c, \qquad (34)$$

$$\tilde{h}_q = M_q(\dot{\tilde{E}}_{P1}^T\dot{\zeta} + \dot{E}_{P11}^T\dot{\zeta}) + h_q.$$

Then, from (29), (33) and (34), the following relation is obtained;

$$\tau = M_q(\tilde{E}_{P1}^T\ddot{\zeta} + E_{P11}^T\ddot{\zeta}) + \tilde{h}_q + \Lambda\tilde{f}$$
$$+ G_{OJ}^+(M_{Or}\tilde{E}_{P2}^T\ddot{\zeta} + \tilde{h}_{Or} - \Xi k_1). \qquad (35)$$

D. Controller

Assume that the desired trajectories $\tilde{\zeta}_d$ and $\tilde{f}_d$ for the movable object position/orientation ζ and the internal force component $\tilde{f}$ are given. From (31), (33) and (34) , the following linearizing control law is considered;

$$\tau = M_W u_p + \Lambda u_I + Q + M_q E_{P11}^T d_{\zeta d} \qquad (36)$$

where

$$M_W = M_q\tilde{E}_{P1}^T + (\tilde{E}_{P2}G_{OJ})^+(\tilde{E}_{P2}M_{Or}\tilde{E}_{P2}^T),$$
$$Q = \tilde{h}_q + (\tilde{E}_{P2}G_{OJ})^+(\tilde{E}_{P2}\tilde{h}_{Or}),$$

and u_p and u_I are new control inputs and correspond to $\ddot{\zeta}$ and $\tilde{f}$, respectively, and $d_{\zeta d}$ denotes an arbitrary constant vector that can provide finger motion that does not affect the object motion. $d_{\zeta d}$ is given such that the fingers meet a certain criterion.

By applying the control law to the system given by (35), we obtain

$$M_q\tilde{E}_{P1}^T(u_p - \ddot{\zeta}) + \Lambda(u_I - \tilde{f}) + M_q E_{P11}^T(d_{\zeta d} - \ddot{\zeta})$$
$$+ (\tilde{E}_{P2}G_{OJ})^+(\tilde{E}_{P2}M_{Or}\tilde{E}_{P2}^T u_p + \tilde{E}_{P2}\tilde{h}_{Or})$$
$$- G_{OJ}^+(M_{Or}\tilde{E}_{P2}^T\ddot{\zeta} + \tilde{h}_{Or} - \Xi k_1) = o. \qquad (37)$$

Here, since $\tilde{E}_{P2}G_{OJ}$ has full row rank, $\text{Im}(\tilde{E}_{P2}^T)\notin \ker(G_{OJ}^T)$, namely $\text{Im}(\tilde{E}_{P2}^T)\in \text{Im}(G_{OJ})$ and then $(G_{OJ}^+)^T G_{OJ}^T \tilde{E}_{P2}^T = \tilde{E}_{P2}^T$. Hence, by multiplying $\tilde{E}_{P2}G_{OJ}$ to the both side of (37) from the left side and using the relation; $G_{OJ}\Lambda = O$, $\tilde{E}_{P2}G_{OJ}(\tilde{E}_{P2}G_{OJ})^+ = I$, $\tilde{E}_{P2}G_{OJ}G_{OJ}^+ = \tilde{E}_{P2}$ and (27), the following relation is obtained;

$$M_{wr}(u_p - \ddot{\zeta}) + M_{wrq}(d_{\zeta d} - \ddot{\zeta}) = o, \qquad (38)$$

where

$$M_{wr} = \tilde{E}_{P2}M_{Or}\tilde{E}_{P2}^T + \tilde{E}_{P2}G_{OJ}M_q\tilde{E}_{P1}^T,$$
$$M_{wrq} = \tilde{E}_{P2}G_{OJ}M_q E_{P11}^T.$$

Since $\tilde{E}_{P2}G_{OJ}$ has full row rank, $\tilde{E}_{P2}G_{OJ}$ can be written by

$$(\tilde{E}_{P2}G_{OJ})^+ = G_{OJ}^T\tilde{E}_{P2}^T(\tilde{E}_{P2}G_{OJ}G_{OJ}^T\tilde{E}_{P2}^T)^{-1}. \qquad (39)$$

From $\text{Im}(E_{P11}^T)\in\ker(J)$, $E_{P11}(\tilde{E}_{P2}G_{OJ})^+ = O$. Hence, by multiplying E_{P11} to the both side of (37)

from the left side and using the relation; $E_{P11}\Lambda = O$ (note that $\text{Im}(\Lambda)\in\text{Im}(J^T)$), $E_{P11}(\tilde{E}_{P2}G_{OJ})^+ = O$, and $E_{P11}G_{OJ}^+ = O$, the following relation is obtained;

$$M_{wqr}(u_p - \ddot{\zeta}) + M_{wq}(d_{\zeta d} - \ddot{\zeta}) = o, \qquad (40)$$

where

$$M_{wqr} = E_{P11}M_q\tilde{E}_{P1}^T,$$
$$M_{wq} = E_{P11}M_q E_{P11}^T.$$

Here, from (3) and (4), we obtain

$$J E_{P1}^T\zeta = G_O^T E_{P2}^T\zeta.$$

From (18) and using the relation; $J E_{P11}^T = O$, this equation is rewritten by

$$J \tilde{E}_{P1}^T\dot{\zeta} = G_O^T\tilde{E}_{P2}^T\dot{\zeta}.$$

Since $\dot{\zeta}$ is arbitrary, we obtain

$$J\tilde{E}_{P1}^T = G_O^T\tilde{E}_{P2}^T.$$

By multiplying J^+ to the both side of this equation from the left side and using the relation; $J^+J\tilde{E}_{P1}^T = \tilde{E}_{P1}^T$ (note that $\text{Im}(\tilde{E}_{P1}^T)\in \text{Im}(J^T)$), we obtain

$$\tilde{E}_{P1}^T = G_{OJ}^T\tilde{E}_{P2}^T.$$

Then, M_{wr} and M_{wqr} become

$$M_{wr} = \tilde{E}_{P2}M_{Or}\tilde{E}_{P2}^T + \tilde{E}_{P2}G_{OJ}M_q G_{OJ}^T\tilde{E}_{P2}^T,$$
$$M_{wqr} = E_{P11}M_q G_{OJ}^T\tilde{E}_{P2}^T = M_{wrq}^T.$$

From (38) and (40), we obtain

$$\begin{pmatrix} M_{wr} & M_{wrq} \\ M_{wrq}^T & M_{wq} \end{pmatrix} \begin{pmatrix} (u_p - \ddot{\zeta}) \\ (d_{\zeta d} - \ddot{\zeta}) \end{pmatrix} \triangleq W\begin{pmatrix} (u_p - \ddot{\zeta}) \\ (d_{\zeta d} - \ddot{\zeta}) \end{pmatrix} = o. \qquad (41)$$

Since M_{wq} is positive definite and then

$$W = W_b\begin{pmatrix} W_a & O \\ O & M_{wq} \end{pmatrix} W_b^T,$$
$$W_a = M_{wr} - M_{wrq}M_{wq}^{-1}M_{wrq}^T,$$
$$W_b = \begin{pmatrix} I & M_{wrq}M_{wq}^{-1} \\ O & I \end{pmatrix},$$

the determinant of W is given by

$$\det W = \det(M_{wq})\det(W_a). \qquad (42)$$

Since M_q is positive definite symmetric, we can define a nonsingular matrix $B_m\in\mathcal{R}^{M\times M}$ such that $M_q = B_m^T B_m$. Letting $B_r^T = \tilde{E}_{P2}G_{OJ}B_m^T$ and $B_q^T = E_{P11}B_m^T$, we obtain

$$W_a = \tilde{E}_{P2}M_{Or}\tilde{E}_{P2}^T + B_r^T(I - B_q(B_q^T B_q)^{-1}B_q^T)B_r.$$

Since both B_r and B_q are of full rank, $(B_q^T B_q)^{-1}B_q^T$ represents B_q^+. Then, the second term of this equation is nonnegative definite. Since $\tilde{E}_{P2}$ is of full row rank, $\tilde{E}_{P2}M_{Or}\tilde{E}_{P2}^T$ is positive definite. Hence, W_a is positive

definite. Therefore, from(42), $\det W \neq 0$. Thus, from (41), we obtain

$$u_p = \ddot{\zeta}, \quad d_{\zeta d} = \ddot{\zeta}. \tag{43}$$

From (39), $\Lambda^T(\tilde{E}_{P2}G_{OJ})^+ = O$. By multiplying Λ^T to the both side of (37) from the left side and using the relation; $\Lambda^T(\tilde{E}_{P2}G_{OJ})^+ = O$ and (43), the following relation is obtained;

$$\Lambda^T\Lambda(u_I - \tilde{f}) = o.$$

Since $\Lambda^T\Lambda$ is nonsingular, we obtain

$$u_I = \tilde{f}. \tag{44}$$

From (43) and (44), the system can be decoupled and linearized by the control law (36). Then, we adopt the following servo controllers with respect to u_p and u_I;

$$u_p = \ddot{\zeta}_d + K_V(\dot{\zeta}_d - \dot{\zeta}) + K_P(\zeta_d - \zeta), \tag{45}$$

$$u_I = \tilde{f}_d + K_I \int_0^t (\tilde{f}_d - \tilde{f})dt' \tag{46}$$

where K_P, K_V and K_I are gain matrices. Then, the object position/orientation error $e_p = \zeta_d - \zeta$ and the force error $e_f = \tilde{f}_d - \tilde{f}$ satisfy the following equations;

$$\ddot{e}_p + K_V\dot{e}_p + K_Pe_p = o, \tag{47}$$

$$\dot{e}_f + K_Ie_f = o. \tag{48}$$

Thus, with appropriate K_P, K_V and K_I, the actual object position/orientation ζ and the component of the internal forces $\tilde{f}$ will converge to the desired trajectories asymptotically.

VII. DETERMINATION OF DESIRED INTERNAL FORCES

In this section, it is shown how to determine the desired magnitudes of internal forces, $\tilde{f}_d$. These magnitudes are determined so that the frictional constraints can be satisfied during the manipulation.

At first, let us consider to formulate internal forces, with respect to contact forces, f. From (6), we get

$$f = G_O^+w_o + \Omega k_3 \tag{49}$$

where $\Omega \in \mathcal{R}^{LD \times (LD-D)}$ is a full column rank matrix whose columns form bases of the null space of G_O and $k_3 \in \mathcal{R}^{(LD-D)}$ is an arbitrary vector expressing the magnitude of each column of Ω.

Here, Ωk_3 expresses the internal forces. However, it is possible that Ωk_3 includes the internal forces which cannot be controlled by the joint torques. Then, in (49), we use $(J^T)^+\Lambda\tilde{f}$, which is the internal forces that can be controlled by the joint torques, in place of Ωk_3;

$$f = G_O^+w_o + (J^T)^+\Lambda\tilde{f}. \tag{50}$$

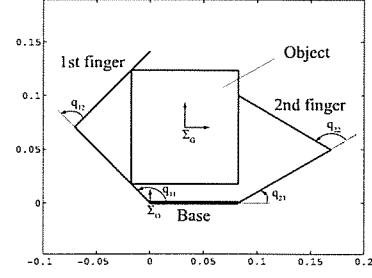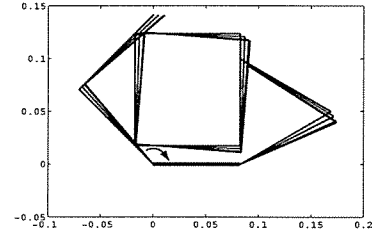Then, assuming the object moves along the desired trajectory, we consider the following problem from (9), (29)
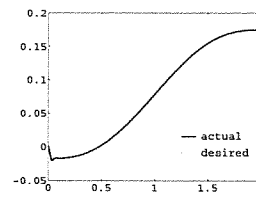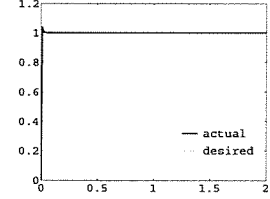

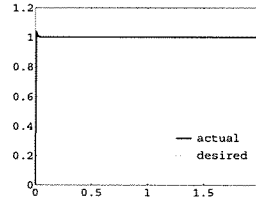Fig. 2. Target System in Simulation (at the initial state)
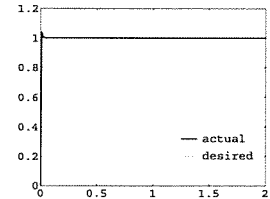

(a) Motion of the object

(b) Trajectory of ϕ (c) Trajectory of $\tilde{f}_1$

(d) Trajectory of $\tilde{f}_2$ (e) Trajectory of $\tilde{f}_3$

Fig. 3. Results

and (50);

$$\min_{\tilde{f}} \quad \rho(\tilde{f})$$

$$\text{subject to} \quad f = G_O^+(w_{O_d} + w_{O_{ext}}) + (J^T)^+\Lambda\tilde{f}$$

$$f \in \mathcal{F}_f$$

$$w_{O_d} = M_{Or}\tilde{E}_{P2}^T\ddot{\zeta}_d + \tilde{h}_{Or} \tag{51}$$

where ρ is a criterion function and $w_{O_{ext}}$ is unexpected external forces and moments. Note that for the practical use, we should better use smaller μ_{ij} in $\mathcal{F}_f$ than the real one. Note also that as a candidate for ρ, we can cite magnitudes of contact forces, magnitudes of joint torques, and so on.

VIII. SIMULATION

In order to verify our approach, we show simulation results. Fig.2 shows the target system in the simulation.

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Σ_O is placed at the root of the 1st finger. Σ_G is placed at the geometrical center of the object. The scale markings shown in Fig.2 represent the ones of Σ_R. Note that when the object is at the initial state, Σ_O corresponds to Σ_R (see Fig.2).

The robotic hand is composed of 2 fingers which are same form. The length of each link is set to 0.1[m]. The mass of each link is set to 0.025[kg]. The object is a $0.075\sqrt{2} \times 0.1$[m] quadrangle. The mass of the object is set to 0.408[kg]. The density of the object is set to be uniform. The contact positions at the initial state are set to $(-0.0177\ 0.0177)$, $(-0.0177\ 0.1237)$, and $(0.0823\ 0.1)$. The servo gains are set to $K_P = \mathrm{diag}(10000.\ 10000.\ 10000.)$ [1/sec^2], $K_V = \mathrm{diag}(100.\ 100.\ 100.)$ [1/sec], and $K_I = \mathrm{diag}(100.\ 100.\ 100.)$ [1/sec]. The sampling time for calculating the input torques is set to 1.0[msec] and the one for calculating the direct dynamics is set to 0.1[msec].

We consider to rotate counterclockwise the object around Σ_O from 0[degree] to 10 [degree]. However, to make an initial error, we rotate the object around Σ_O from -1[degree] to 10 [degree] in the desired trajectory. The frictional coefficients at all contact points are set to 0.3. The desired magnitudes of internal forces are set to $(1\ 1\ 1)$[N] which can satisfy the constraints of the problem given by (51).

Fig.3 shows the results of the simulation. Fig.3 (a) shows the outline of the motion of the grasped object. Fig.3 (b) shows the trajectories of desired and actual ϕ's that corresponds to the orientation of the object. Fig.3 (c)~(e) show the trajectories of desired and actual $\bar{f}_i$'s ($i = 1, 2, 3$) that corresponds to the magnitudes of internal forces. From Fig.3, we can see that the ϕ and $\bar{f}_i$'s ($i = 1, 2, 3$) all converge to their desired trajectories, although there are initial errors.

IX. CONCLUSION

In this paper, we have proposed a control algorithm for manipulating an object under hybrid active/passive closure. For the purpose, we have shown the orthogonality between the directions of active and passive force closures for general grasping systems. By using the orthogonality, we have decomposed the object dynamics into the part corresponding to active force closure and the part corresponding to passive force closure. By using the decomposition, we have shown that we have only to take into consideration the dynamics of the active part in order to derive a control algorithm. In order to satisfy the frictional constraints, we also have shown a way for determining desired internal forces. The simulation results show the effectiveness of our approach.

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