A Note on Bottom-Up Pyramid Acceptors

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Abstract

In this paper, we investigate the relationship between the accepting powers of deterministic bottom-up pyramid acceptors and deterministic two-dimensional finite automata, and show that there is a set accepted by a deterministic two-dimensional finite automaton but not by any deterministic bottom-up pyramid acceptor which operates in time of order lower than the diameter of the input, and vice versa.

1. Introduction

In [1], Dyer and Rosenfeld introduced a new type of acceptor on a two-dimensional pattern (or tape), called the "pyramid cellular acceptor" (denoted by "PA"), and demonstrated that many useful recognition tasks are executed by PA's in time proportional to the logarithm of the diameter of the input.

They also introduced a "bottom-up pyramid acceptor" (denoted by "BPA") which is a restricted version of the PA, and proposed some interesting open problems about BPA's. We are interested in the following problem (which is one of the open problems): Does the class of sets accepted by deterministic BPA's include the class of sets accepted by deterministic two-dimensional finite automata (denoted by "2–DA's") [2, 3, 4]?

In this note, we show that the class of sets accepted by 2–DA's is incomparable with the class of sets accepted by deterministic BPA's which operate in time of order lower than the diameter of the input.

2. Preliminaries

We first give some definitions and notations concerning two-dimensional tapes.

Definition 1: Let Σ be a finite set of symbols. A *two-dimensional tape* over Σ is a two-dimensional rectangular array of elements of Σ.

The set of all two-dimensional tapes over Σ is denoted by $\Sigma^{(2)}$. Given a tape $x \in \Sigma^{(2)}$, we let $l_1(x)$ be the number of rows of x and $l_2(x)$ be the number of columns of x. If $1 \leq i \leq l_1(x)$ and $1 \leq j \leq l_2(x)$, we let $x(i, j)$ denote the symbol in x with coordinates (i, j). Furthermore, we define

$$x[(i, j), (i', j')]$$

only when $1 \leq i \leq i' \leq l_1(x)$ and $1 \leq j \leq j' \leq l_2(x)$ as the two-dimensional tape z satisfying

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the following (i) and (ii):

(i) $l_1(z)=i'-i+1$ and $l_2(z)=j'-j+1$;

(ii) for each k, r $(1\leq k\leq l_1(z),\ 1\leq r\leq l_2(z))$,

$$z(k, r)=x(k+i-1, r+j-1).$$

We next review some basic concepts about bottom-up pyramid acceptors (BPA's). A *bottom-up pyramid acceptor* is a pyramid stack of two-dimensional cellular arrays, where the bottom array has size 2^n by 2^n $(n\geq 1)$, the next lowest 2^{n-1} by 2^{n-1}, and so forth, the $(n+1)$st layer consisting of a single cell, called the *root*. Each cell is defined as an identical finite state machine, $M=(Q_N, Q_T, \delta, A)$, where Q_N is a nonempty, finite set of *states*, $Q_T\subseteq Q_N$ is a finite set of *input states*, $A\subseteq Q_N$ is the set of *accepting states, and* δ: $Q_N^5\to Q_N$ is the *state transition function*, mapping the current state of M and its four son cells in a 2-by-2 block in the level below into M's next state. As shown in Fig. 1, let c be some cell in the $(i+1)$st layer $(i\geq 1)$, and let c_1, c_2, c_3 and c_4 be four son cells (in the i-th layer) of c. Then

$$q_c(t+1)=\delta(q_c(t), q_{c_1}(t), q_{c_2}(t), q_{c_3}(t), q_{c_4}(t)),$$

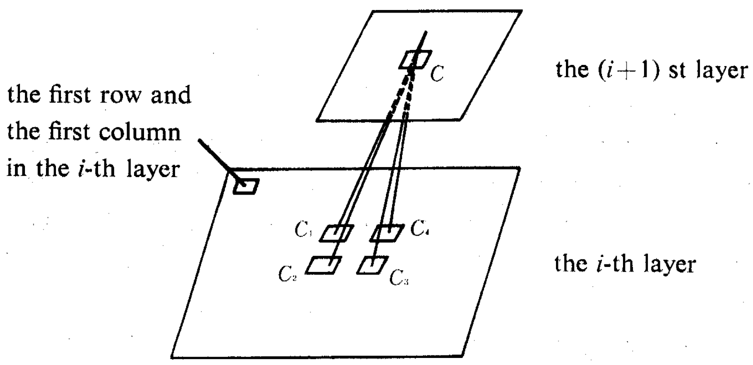

Fig. 1

where for example $q_c(t)$ means the state of c at time t. (Especially, we assume without loss of generality that the next state of each cell c' in the bottom array depends only on the current state of c'.)

At time $t=0$, the input tape $x\in Q_T^{(2)}$ $(l_1(x)=l_2(x)=2^n,\ n\geq 1)$ is stored as the initial states of the bottom array (in such a way that $x(i, j)$ is stored at the cell of the i-th row and the j-th column), and the other cells are initialized to a *quiescent state* "q_s" $(\in Q_N-Q_T-A)$. As usual, we let $\delta(q_s, q_s, q_s, q_s, q_s)=q_s$.

The input is *accepted* if and only if the root cell ever enters an accepting state. This BPA is called *deterministic*. A *nondeterministic* bottom-up pyramid acceptor (NBPA) is defined as a BPA using δ: $Q_N^5\to 2^{Q_N}$ instead of the state transition function of the deterministic BPA. Below, we denote the deterministic BPA by "DBPA", and the nondeterministic BPA by "NBPA".

A DBPA (or NBPA) *operates in time* $T(n)$ if for every tape of size 2^n by 2^n $(n\geq 1)$ it accepts, there is an accepting computation which uses no more than time $T(n)$.

Especially, we say that a DBPA (or NBPA) *operates in real time* if it operates in time $T(n)=n$. Let $\mathscr{L}$[DBPA($T(n)$)] ($\mathscr{L}$[NBPA($T(n)$)] denote the class of sets accepted by DBPA's (NBPA's) which operate in time $T(n)$.

3. Results

In this section, we show that the class of sets accepted by deterministic two-dimensional finite automata (2−DA's)‡ is incomparable with the class of sets accepted by DBPA's which operates in time of order lower than the diameter of the input. We assume, in this paper, that the sizes of inputs to automata are restricted to 2^n by 2^n $(n\geq 1)$.

Lemma 1: Let $U=\{x\in\{0, 1\}^{(2)}|\exists n(n\geq 1)[l_1(x)=l_2(x)=2^n]$ and $x(2^{n-1}, 2^{n-1})=1\}$. Then,

(1) $U\notin\mathscr{L}[2-\mathrm{DA}]$, and

(2) $U\in\mathscr{L}[\mathrm{DBPA}(n)]$.

Proof. The proof of (1) is similar to that of Theorem 1 in [2]. So the proof is omitted here. Below, we prove (2). The set U is accepted by the following DBPA B which operates in real time. Each cell of B is defined as the following finite state machine $M=(Q_N, Q_T, \delta, A)$:

(i) $Q_N=\{q_s, 0, 1\}\cup\{Y, N\}\times\{0, 1\}$, where q_s is the quiescent state;

(ii) $Q_T=\{0, 1\}$;

(iii) $A=\{[Y, 1], [Y, 0]\}$;

(iv) ① For any $a, b, c\in\{0, 1\}$,

$\delta(q_s, 1, a, b, c)=[Y, b]$, and

$\delta(q_s, 0, a, b, c)=[N, b]$;

② For any $p, q, r\in\{Y, N\}$, and for any $a, b, c\in\{0, 1\}$,

$\delta(q_s, [Y, 1], [p, a], [q, b], [r, c])=[Y, b]$,

$\delta(q_s, [N, 1], [p, a], [q, b], [r, c])=[Y, b]$,

$\delta(q_s, [Y, 0], [p, a], [q, b], [r, c])=[N, b]$, and

$\delta(q_s, [N, 0], [p, a], [q, b], [r, c])=[N, b]$.

It is straightforward to see that B accepts U, and so (2) of the lemma holds. Q. E. D.

Lemma 2: Let $V=\{x\in\{0, 1\}^{(2)}|\exists n(n\geq 1)[l_1(x)=l_2(x)=2^n]$ and $x[(1, 1), (1, 2^n)]=x[(2^n, 1), (2^n, 2^n)]\}$. Let $T(n)$ be a time function such that $\lim_{n\to\infty}[T(n)/2^n]=0$. Then,

(1) $V\in\mathscr{L}[2-\mathrm{DA}]$, and

(2) $V\notin\mathscr{L}[\mathrm{DBPA}(T(n))]$.

Proof: It is obvious that there is a 2−DA accepting V, and so (1) of the Lemma holds. Below, we prove (2). Suppose that there is a DBPA B which accepts V and operates in time $T(n)$, and that each cell of B has k states. For each $n\geq 2$, let

‡ See [2, 3, 4] for definitions of 2-DA's.

$$W(n)=\{x\in\{0,1\}^{(2)}|l_1(x)=l_2(x)=2^n\}, \quad \text{and}$$

$$W'(n)=\{x\in\{0,1\}^{(2)}|l_1(x)=l_2(x)=2^{n-1}$$
$$\& \ x[(1,1),(1,2^{n-1})]\in\{0,1\}^{(2)}$$
$$\& \ x[(2,1),(2^{n-1},2^{n-1})]\in\{0\}^{(2)}\}.$$

We consider the cases when the tapes in $W(n)$ are presented to B. Let c be the cell which is situated at the first row and the first column in the n-th layer (i.e., the layer just below the root cell). (Note that there are four cells in the n-th layer.) For each x in $W(n)$ such that $x[(1,1),(2^{n-1},2^{n-1})]\in W'(n)$, and for each $r\geq 1$, let $q_r(x)$ be the state of c at time r when x is presented to B. Then, the following proposition must hold.

Proposition 1: Let x, y be two different tapes in $W(n)$ such that both $x[(1,1),(2^{n-1},2^{n-1})]$ and $y[(1,1),(2^{n-1},2^{n-1})]$ are in $W'(n)$ and $x[(1,1),(2^{n-1},2^{n-1})]\neq y[(1,1),(2^{n-1},2^{n-1})]$. Then,

$$<q_1(x), q_2(x),\dots, q_{T(n)}(x)> \neq <q_1(y), q_2(y),\dots, q_{T(n)}(y)>.$$

[For suppose that $<q_1(x), q_2(x),\dots, q_{T(n)}(x)> = <q_1(y), q_2(y),\dots, q_{T(n)}(y)>$. We consider two tapes z, z' in $W(n)$ such that

(i) $z[(1,1),(2^{n-1},2^{n-1})]=x[(1,1),(2^{n-1},2^{n-1})]$ and $z'[(1,1),(2^{n-1},2^{n-1})]=y[(1,1),(2^{n-1},2^{n-1})]$,

(ii) the part of z except for $z[(1,1),(2^{n-1},2^{n-1})]$ is identical with the part of z' except for $z'[(1,1),(2^{n-1},2^{n-1})]$, and

(iii) $z[(1,1),(1,2^n)]=z[(2^n,1),(2^n,2^n)]$.

By assumption, the root cell of B enters the same states until time $T(n)$, for the tapes z and z'. Since B operates in time $T(n)$ and z is in V, it follows that z' is also accepted by B. This contradicts the fact that z' is not in V.] Let $t(n)$ be the number of different sequences of states which c enters until time $T(n)$. Clearly, $t(n)\leq k^{T(n)}$. On the other hand, $|W'(n)|^{\ddagger}=2^{2^{n-1}}$. Since $\lim_{n\to\infty}[T(n)/2^n]=0$ (by assumption of the lemma), it follows that $|W'(n)|>t(n)$ for large n. Therefore, it follows that for large n there must exist two different tapes x, y in $W(n)$ such that

(i) both $x[(1,1),(2^{n-1},2^{n-1})]$ and $y[(1,1),(2^{n-1},2^{n-1})]$ are in $W'(n)$,

(ii) $x[(1,1),(2^{n-1},2^{n-1})]\neq y[(1,1),(2^{n-1},2^{n-1})]$, and

(iii) $<q_1(x), q_2(x),\dots, q_{T(n)}(x)> = <q_1(y), q_2(y),\dots, q_{T(n)}(y)>$.

This contradicts the above Proposition 1, and thus the part (2) of the lemma holds.

Q.E.D.

From lemmas 1 and 2, we can get the following theorem.

Theorem 1: Let $T(n)$ be a time function such that $\lim_{n\to\infty}[T(n)/2^n]=0$ and $T(n)\geq n$ $(n\geq 1)$. Then $\mathscr{L}[2-\text{DA}]$ is incomparable with $\mathscr{L}[\text{DBPA}(T(n))]$.

‡ For any set S, let |S| denote the number of elements of S.

Corollary 1: $\mathscr{L}[2-\mathrm{DA}]$ is incomparable with $\mathscr{L}[\mathrm{DBPA}(n)]$, which is the class of sets accepted by DBPA's operating in real time.

For any NBPA M which operates in real time, we can construct a DBPA which accepts the same set as M and operates in real time, by using the well-known subset construction method. Thus $\mathscr{L}[\mathrm{NBPA}(n)]=\mathscr{L}[\mathrm{DBPA}(n)]$. From this and Corollary 1, we get the following corollary.

Corollary 2: $\mathscr{L}[2-\mathrm{DA}]$ is incomparable with $\mathscr{L}[\mathrm{NBPA}(n)]$.

It is still unknown whether the class of sets accepted by DBPA's includes $\mathscr{L}[2-\mathrm{DA}]$.

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References

[1] C. R. Dyer and A. Rosenfeld: "Cellular pyramids for image analysis," University of Maryland, Computer Science Center, TR-**544**, AFOSR-77-3271.

[2] M. Blum and C. Hewitt: "Automata on a two-dimensional tape," IEEE Symp. on SWAT (1967), p. 155.

[3] K. Inoue and A. Nakamura: "Some properties of two-dimensional on-line tessellation acceptors, Information Sciences **13**, 95–121 (1977).

[4] K. Inoue, I. Takanami and A. Nakamura: "A note on two-dimensional finite automata," Information Processing Letters **7**, 1, January 1978, p. 49.