Studies on Reactive Mufflers
(Part 1, Fundamental Formulas for the Attenuation)

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Abstract

This paper presents theoretical formulas for the attenuation of reactive mufflers designed to reduce the noise radiating into free space.

The values calculated from the above formulas agree fairly well with those obtained from the experiments.

1. Theoretical study (Constant velocity sound source)

In the derivation of the equations for the attenuation of reactive mufflers, the following conditions are assumed:

(1) The sound pressures are small compared with the absolute value of the average pressure in the system.

(2) The muffler walls neither conduct nor transmit sound energy.

(3) Flow and viscosity effects may be neglected.

(4) The plane sound waves need be considered.

The cavity appearing in Fig. 1 is the type most commonly used in mufflers. In this figure, l_i and l_0 are the lengths of the inlet and the outlet pipes, respectively. And l_{i1} and l_{01} are the inserted lengths of the inlet and the outlet pipes, respectively.

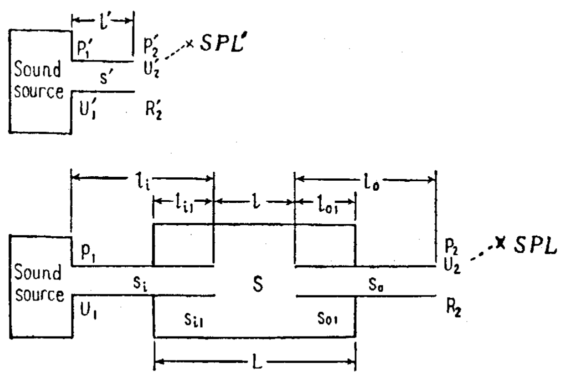

Fig. 1 General shape of a reactive muffler.

Without a muffler attached, the matrix of pipe l' is expressed as

$$\begin{bmatrix} p'_1 \\ U'_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} p'_2 \\ U'_2 \end{bmatrix} \qquad \cdots\cdots\cdots(1)$$

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and with an attached muffler, the matrix of the entire pipe system as

$$\begin{bmatrix} p_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_2 \\ U_2 \end{bmatrix} \qquad \cdots\cdots\cdots(2)$$

where, p denotes the effective value of sound pressure, U the effective value of volume velocity, suffix 1 the inlet, and suffix 2 the outlet.

When the sound source is the constant velocity source, i.e., when the volume velocity is the same with or without a muffler attached,

$$U_i' = U_1 \qquad \cdots\cdots\cdots(3)$$

When ka ($k=2\pi f/c$, f is frequency, c is the speed of sound and a is the radius of the outlet) is less than 0.6, the attenuation (dB) is expressed as follows:

$$Att = 10\log_{10}\frac{\text{Radiation power at outlet without a muffler}}{\text{Radiation power at outlet with a muffler}}$$

$$= 20\log_{10}\frac{U_2'}{U_2} + 10\log\frac{R'}{R}$$

$$= 20\log_{10}\frac{U_1'/D'}{U_1/D} + 10\log\frac{R'}{R} \qquad \cdots\cdots\cdots(4)$$

where R and R' are respectively radiation resistance at the outlet with and without a muffler attached. The value of $10\log_{10} R'/R$ is negligibly small.

If Eq. (3) holds true and R' is equal to R, the attenuation can be expressed as

$$Att = 20\log_{10} D - 20\log_{10} D' \qquad \cdots\cdots\cdots(5)$$

Thus, the Att for the system in Fig. 1 can be determined if the matrix of the entire pipe system is formulated and D' in Eq. (1) and D in Eq. (2) are found. The most effective approach to finding the matrix of the pipe system is to simulate this system with electrical circuit. Simulation yields the diagram appearing in Fig. 2, and the matrix is as follows:

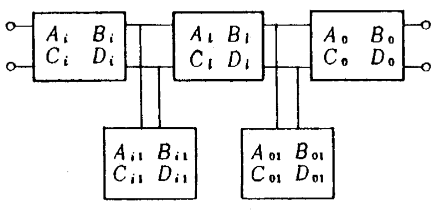

Fig. 2 Equivalent circuit of Fig. 1.

$$\begin{bmatrix} p_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \begin{bmatrix} A_{i1} & B_{i1} \\ C_{i1} & D_{i1} \end{bmatrix} \begin{bmatrix} A_l & B_l \\ C_l & D_l \end{bmatrix}$$

$$\times \begin{bmatrix} A_{01} & B_{01} \\ C_{01} & D_{01} \end{bmatrix} \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} \begin{bmatrix} p_2 \\ U_2 \end{bmatrix} \qquad \cdots\cdots\cdots(6)$$

where i is the inlet pipe, $i1$ that portion of the inlet pipe inserted, 01 that portion of the outlet pipe inserted, 0 the outlet pipe, and l that cavity length without insertion.

When both ends of a tube l_x are open, the matrix can be expressed as follows:

$$\begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} = \begin{bmatrix} \cos kl_x & j\frac{\rho c}{S_x}\sin kl_x \\ j\frac{S_x}{\rho c}\sin kl_x & \cos kl_x \end{bmatrix} \quad \cdots\cdots\cdots(7)$$

where S_x is the cross-sectional area and j is $\sqrt{-1}$.

When one end of a tube l_x is closed, the matrix is expressed as follows:

$$\begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\frac{S_x}{\rho c}\tan kl_x & 1 \end{bmatrix} \quad \cdots\cdots\cdots(8)$$

By substituting Eqs. (7) and (8) in Eq. (6):

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos kl_i & j\frac{\rho c}{S_i}\sin kl_i \\ j\frac{S_i}{\rho c}\sin kl_i & \cos kl_i \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j\frac{S_{i1}}{\rho c}\tan kl_{i1} & 1 \end{bmatrix} \begin{bmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{\rho c}{S}\sin kl & \cos kl \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 0 \\ j\frac{S_{01}}{\rho c}\tan kl_{01} & 1 \end{bmatrix} \begin{bmatrix} \cos kl_0 & j\frac{\rho c}{S_0}\sin kl_0 \\ j\frac{S_0}{\rho c}\sin kl_0 & \cos kl_0 \end{bmatrix} \quad \cdots\cdots\cdots(9)$$

We can obtain D by expanding the right side of the above equation.

And D' is:

$$D' = \cos kl' \quad \cdots\cdots\cdots(10)$$

By substituting D and D' in Eq. (5), the following formula is obtained.

$$Att = 20\log_{10}\Big|\cos kl_i \cos kl \cos kl_0 - \frac{S_i}{S}\sin kl_i \sin kl \cos kl_0 - \frac{S_i}{S_0}\sin kl_i \cos kl \sin kl_0$$
$$-\frac{S}{S_0}\cos kl_i \sin kl \sin kl_0 - \frac{S_i}{S}\frac{S_{01}}{S_0}\sin kl_i \sin kl \tan kl_{01} \sin kl_0$$
$$-\frac{S_{i1}}{S_0}\cos kl_i \tan kl_{i1} \cos kl \sin kl_0 - \frac{S_{i1}}{S}\cos kl_i \tan kl_{i1} \sin kl \cos kl_0$$
$$-\frac{S_{01}}{S_0}\cos kl_i \cos kl \tan kl_{01} \sin kl_0$$
$$+\frac{S_{i1}}{S}\frac{S_{01}}{S_0}\cos kl_i \tan kl_{i1} \sin kl \tan kl_{01} \sin kl_0\Big| - 20\log_{10}|\cos kl'| \ \text{dB}$$
$$\cdots\cdots\cdots(11)$$

It should be noted that each pipe length value includes end-correction at both pipe ends. This is a precise equation for the attenuation effects of a single-stage reactive muffler connected to a constant velocity sound source, in which S_{i1} and S_{01} represent the difference in cross-sectional areas between the cavity and the inlet pipe, and the cavity and the outlet pipe respectively.

When the diameters of both pipes, inlet and outlet, are indentical, Eq. (11) can be simplified.

By assuming the following relationship,

$$S'=S_i=S_0=s, \quad S_{i1}=S_{01}\fallingdotseq S \qquad \cdots\cdots\cdots(12)$$

and substituting Eq. (12) in Eq. (11), the expression becomes as follows:

$$\begin{aligned} Att=20\log_{10}\Big|&\cos kl_i\cos kl_0(\cos kl-\tan kl_{i1}\sin kl)\\ &-\frac{S}{s}\cos kl_i\sin kl_0(\sin kl+\tan kl_{i1}\cos kl+\cos kl\tan kl_{01}\\ &-\tan kl_{i1}\sin kl\tan kl_{01})-\sin kl_i\sin kl_0(\cos kl-\sin kl\tan kl_{01})\\ &-\frac{s}{S}\sin kl_i\sin kl\cos kl_0\Big|-20\log_{10}|\cos kl'| \quad \text{dB} \qquad \cdots\cdots\cdots(13)\end{aligned}$$

i.e.,

$$\begin{aligned} Att=20\log_{10}\Big|&\frac{\cos kl_i\cos k(l_{i1}+l)\cos kl_0}{\cos kl_{i1}}-\frac{S}{s}\frac{\cos kl_i\sin kL\sin kl_0}{\cos kl_{i1}\cos kl_{01}}\\ &-\frac{\sin kl_i\cos k(l+l_{01})\sin kl_0}{\cos kl_{01}}-\frac{s}{S}\sin kl_i\sin kl\cos kl_0\Big|\\ &-20\log_{10}|\cos kl'| \quad \text{dB} \qquad \cdots\cdots\cdots(14)\end{aligned}$$

Equation (14) is a general formula for the attenuation of the muffler connected to a constant velocity sound source when the diameters of the inlet and outlet pipes are the same, and from it many *Att* equations can be derived.

Some examples of derivation from Eq. (14) are as follows:

i) When $l'=l_i$

$$\begin{aligned} Att=20\log_{10}\Big|&\frac{\cos k(l_{i1}+l)\cos kl_0}{\cos kl_{i1}}-\frac{S}{s}\frac{\sin kL\sin kl_0}{\cos kl_{i1}\cos kl_{01}}\\ &-\frac{\tan kl_i\cos k(l+l_{01})\sin kl_0}{\cos kl_{01}}-\frac{s}{S}\tan kl_i\sin kl\cos kl_0\Big| \quad \text{dB}\end{aligned}$$

$$\cdots\cdots\cdots(15)$$

ii) When $l'=l_i$ and $l_{i1}=0$

$$Att=20\log_{10}\Big|\cos kl\cos kl_0-\frac{\sin kl_0}{\cos kl_{01}}\Big(\frac{S}{s}\sin kL+\tan kl_i\cos kL\Big)$$

$$-\frac{s}{S}\tan kl_i \sin kl \cos kl_0\Big| \quad \mathrm{dB} \qquad \cdots\cdots\cdots(16)$$

iii) When $l'=0$ and $l_{i1}=l_{01}=0$

$$Att=20\log_{10}\Big|\cos kl_i \cos kL \cos kl_0-\frac{S}{s}\cos kl_i \sin kL \sin kl_0$$

$$-\sin kl_i \cos kL \sin kl_0-\frac{s}{S}\sin kl_i \sin kL \cos kl_0\Big| \quad \mathrm{dB} \qquad \cdots\cdots\cdots(17)$$

iv) When $l'=l_i$ and $l_{i1}=l_{01}=0$

$$Att=20\log_{10}\Big|\cos kL \cos kl_0-\frac{S}{s}\sin kL \sin kl_0$$

$$-\tan kl_i \cos kL \sin kl_0-\frac{s}{S}\tan kl_i \sin kL \cos kl_0\Big| \quad \mathrm{dB} \qquad \cdots\cdots\cdots(18)$$

v) When $l'=l_i=0$, $l_{i1}=l_{01}=0$

$$Att=20\log_{10}\Big|\cos kL \cos kl_0-\frac{S}{s}\sin kL \sin kl_0\Big| \quad \mathrm{dB} \qquad \cdots\cdots\cdots(19)$$

vi) Approximate equation, when $l'=l_i$

$$Att \fallingdotseq 20\log_{10}\left|\frac{S}{s}\,\frac{\sin kL \sin kl_0}{\cos kl_{i1}\cos kl_{01}}\right| \quad \mathrm{dB} \qquad \cdots\cdots\cdots(20)$$

Equation (20) is approximate except within the range where $|\tan kl_i|$ increases substantially and within the low frequency range deemed as a lumped constant.

vii) Approximate equation, when $l_{i1}=l_{01}=0$
If $l_{i1}=l_{01}=0$ and $l' \neq l_i$ in Eq. (14), Att is:

$$Att=20\log_{10}\Big|\cos kl_i \cos kL \cos kl_0-\frac{S}{s}\cos kl_i \sin kL \sin kl_0$$

$$-\sin kl_i \cos kL \sin kl_0-\frac{s}{S}\sin kl_i \sin kL \cos kl_0\Big|-20\log_{10}|\cos kl'| \quad \mathrm{dB}$$

$$\cdots\cdots\cdots(21)$$

Moreover, if $l'=l_i$, Att is:

$$Att=20\log_{10}\Big|\frac{S}{s}\sin kL \sin kl_0\Big| \quad \mathrm{dB} \qquad \cdots\cdots\cdots(22)$$

This is approximate except for the low frequency range deemed as a lumped constant. But note that although this is true when $l'=l_i=0$, errors increases where l' (i.e. l_i) becomes longer and the values of $|\tan kl_i|$ (i.e. $|\tan kl'|$) increase.

2. Theoretical study (Constant pressure sound source)

When the sound source is constant pressure, i.e., the pressure is not variable with or without a muffler attached,

$$p_1' = p_1 \qquad \cdots\cdots\cdots(23)$$

Thus, similar to Eq. (4), *Att* is derived as follows:

$$Att = 20\log_{10} U_2'/U_2 + 10\log_{10} R'/R = 20\log_{10}\frac{p_1'/B'}{p_1/B} + 10\log_{10} R'/R \qquad \cdots\cdots\cdots(24)$$

and if Eq. (23) holds true and R' is equal to R, Eq. (24) can be expressed as

$$Att = 20\log_{10} B - 20\log_{10} B' \qquad \cdots\cdots\cdots(25)$$

where B is obtained by expanding the right side of Eq. (9).

On the other hand, B' is:

$$B' = \frac{j\rho c}{S'}\sin kl' \qquad \cdots\cdots\cdots(26)$$

By substituting B and B' in Eq. (25), *Att* becomes:

$$\begin{aligned} Att = 20\log_{10}\Big| & \frac{S'}{S}\cos kl_i \sin kl \cos kl_0 + \frac{S'}{S_i}\sin kl_i \cos kl \cos kl_0 \\ & + \frac{S'}{S_0}\cos kl_i \cos kl \sin kl_0 - \frac{S'}{S_0}\frac{S}{S_i}\sin kl_i \sin kl \sin kl_0 \\ & - \frac{S'}{S_0}\frac{S_{01}}{S}\cos kl_i \sin kl \tan kl_{01} \sin kl_0 - \frac{S'}{S_0}\frac{S_{i1}}{S_i}\sin kl_i \tan kl_{i1}\cos kl \sin kl_0 \\ & - \frac{S'}{S_0}\frac{S_{01}}{S_i}\sin kl_i \cos kl \tan kl_{01}\sin kl_0 - \frac{S'}{S}\frac{S_{i1}}{S_i}\sin kl_i \tan kl_{i1}\sin kl \cos kl_0 \\ & + \frac{S'}{S_0}\frac{S_{i1}}{S_i}\frac{S_{01}}{S}\sin kl_i \tan kl_{i1}\sin kl \tan kl_{01}\sin kl_0\Big| - 20\log_{10}|\sin kl'| \quad \text{dB} \end{aligned} \qquad \cdots\cdots\cdots(27)$$

Equation (27) is a precise formula giving the attenuation a reactive muffler is connected to a constant pressure sound source as shown in Fig. 1.

This formula is fairly complicated, but by substituting the conditions of Eq. (12) in Eq. (27), *Att* becomes as follows:

$$\begin{aligned} Att = 20\log_{10}\Big|\frac{1}{s}\Big\{ & \sin kl_i \cos kl_0(\cos kl - \tan kl_{i1}\sin kl) - \frac{S}{s}\sin kl_i \sin kl_0 \\ & \times(\sin kl + \tan kl_{i1}\cos kl + \cos kl \tan kl_{01} - \tan kl_{i1}\sin kl \tan kl_{01}) \end{aligned}$$

$$+\cos kl_i \sin kl_0(\cos kl - \sin kl \tan kl_{01})$$

$$+\frac{s}{S}\cos kl_i \sin kl \cos kl_0\Big\}\Big| - 20\log_{10}\left|\frac{1}{s}\sin kl'\right| \quad \text{dB} \qquad \cdots\cdots\cdots(28)$$

i.e.,

$$Att = 20\log_{10}\left|\frac{\sin kl_i \cos k(l_{i1}+l)\ cns\ kl_0}{\cos kl_{i1}} - \frac{S}{s}\frac{\sin kl_i \sin kL \sin kl_0}{\cos kl_{i1}\cos kl_{01}}\right.$$

$$\left. + \frac{\cos kl_i \cos k(l+l_{01}) \sin kl_0}{\cos kl_0} + \frac{s}{S}\cos kl_i \sin kl \cos kl_0\right|$$

$$-20\log_{10}|\sin kl'| \quad \text{dB} \qquad \cdots\cdots\cdots(29)$$

Equation (29) is a general expression for the attenuation of the reactive muffler connected to a constant pressure sound source when the diameters of the inlet and outlet pipe are the same.

From Eq. (29), the derivations below can be obtained under various conditions.

i) When $l' = l_i$

$$Att = 20\log_{10}\left|\frac{\cos k(l_{i1}+l)\cos kl_0}{\cos kl_{i1}} - \frac{S}{s}\frac{\sin kL \sin kl_0}{\cos kl_{i1}\cos kl_{01}}\right.$$

$$\left. + \frac{\cot kl_i \cos k(l+l_{01}) \sin kl_0}{\cos kl_{01}} + \frac{s}{S}\cot kl_i \sin kl \cos kl_0\right| \quad \text{dB}$$

$$\cdots\cdots\cdots(30)$$

ii) When $l' = l_i$ and $l_{i1} = 0$

$$Att = 20\log_{10}\left|\cos kl \cos kl_0 - \frac{\sin kl_0}{\cos kl_{01}}\left(\frac{S}{s}\sin kL - \cot kl_i \cos kL\right)\right.$$

$$\left. + \frac{s}{S}\cot kl_i \sin kl \cos kl_0\right| \quad \text{dB} \qquad \cdots\cdots\cdots(31)$$

iii) When $l' = l_i$ and $l_{i1} = l_{01} = 0$

$$Att = 20\log_{10}\left|\cos kL \cos kl_0 - \frac{S}{s}\sin kL \sin kl_0\right.$$

$$\left. + \cot kl_i \cos kL \sin kl_0 + \frac{s}{S}\cot kl_i \sin kL \cos kl_0\right| \quad \text{dB} \qquad \cdots\cdots\cdots(32)$$

iv) Approximate equation, when $l' = l_i$

For approximate purposes, excepting the range in which $|\cot kl_i|$ increases substantially and the low-frequency range is deemed as a lumped constant,

$$Att \fallingdotseq 20\log_{10}\left|\frac{S}{s}\frac{\sin kL \sin kl_0}{\cos kl_i \cos kl_{01}}\right| \quad \text{dB} \qquad \cdots\cdots\cdots(33)$$

v) Approximate equation, when $l_{i1} = l_{01} = 0$

If $l_{i1}=l_{01}=0$ and $l' \neq l_i$ in Eq. (29), Att is:

$$Att=20\log_{10}\Big|\sin kl_i \cos kL \cos kl_0 - \frac{S}{s}\sin kl_i \sin kL \sin kl_0 + \cos kl_i \cos kL \sin kl_0 + \frac{s}{S}\cos kl_i \sin kL \cos kl_0\Big| - 20\log_{10}|\sin kl'| \quad \text{dB} \qquad \cdots\cdots\cdots(34)$$

Moreover, if $l'=l_i$, Att is:

$$Att \fallingdotseq 20\log_{10}\Big|\frac{S}{s}\sin kL \sin kl_0\Big| \quad \text{dB} \qquad \cdots\cdots\cdots(35)$$

As already mentioned, this is approximate, and applicable except where the values of $|\cot kl_i|$ increase substantially and the low-frequency range is deemed as a lumped constant.

3. Approximate equations for the low-frequency range

Calculations using approximate equations such as Eqs. (20) and (33) result in errors in the value of Att in the vicinity of the system's resonance frequency, and thus in the lower frequency ranges it is necessary to use exact and more applicable equations such as Eq. (15) and the expression, (30). Howerer, in this case, the computation becomes very complex.

One solution is as follows. As in the case of the exhaust system of an automobile, if a small cavity is attached to the end of a long exhaust pipe and the wave length is large relative to the dimensions, when $l'=l_i$ the approximate attenuation can be determined by assuming $\sin kl_i \fallingdotseq kl_x$ and $\cos kl_x \fallingdotseq 1$, as in Eqs. (15) and (30):

$$Att \fallingdotseq 20\log_{10}\Big|1-k^2\frac{Vl_0}{s}+k\Big(\frac{s}{S}l+l_0\Big)\zeta\Big| \quad \text{dB} \qquad \cdots\cdots\cdots(36)$$

where V is cavity capacity and ζ is as follows:

$$\left.\begin{aligned} \zeta &= -\tan kl_i \ \text{(constant velocity sound source)} \\ \zeta &= \cot kl_i \ \text{(constant pressure sound source)} \end{aligned}\right\} \qquad \cdots\cdots\cdots(37)$$

3.1 Constant velocity sound source

From Eq. (37) in the frequency range where the wave length is far greater than l_i,

$$\zeta \fallingdotseq -kl_i \qquad \cdots\cdots\cdots(38)$$

thus,

$$Att=20\log_{10}\Big|1-k^2\frac{Vl_0}{s}-k^2\Big(\frac{s}{S}l+l_0\Big)l_i\Big| \quad \text{dB} \qquad \cdots\cdots\cdots(39)$$

Here, since $S \gg s$, $k\left(\frac{s}{S}l+l_0\right) \ll 1$.
If the third term is disregarded, Att can be approximated

$$Att = 20\log_{10}\left|1-k^2\frac{Vl_0}{s}\right| \quad \mathrm{dB} \qquad \cdots\cdots\cdots(40)$$

This equation is relatively well known.

3.2 Constant pressure sound source

From Eq. (37) in the frequency range where the wave length is far greater than l_i,

$$\zeta \doteqdot \frac{1}{kl_i} \qquad \cdots\cdots\cdots(41)$$

hence,

$$Att = 20\log_{10}\left|1-k^2\frac{Vl_0}{s}+\left(\frac{s}{S_i}\frac{l}{l_i}+\frac{l_0}{l_t}\right)\right| \quad \mathrm{dB} \qquad \cdots\cdots\cdots(42)$$

If $\frac{s}{S}\frac{l}{l_i} \ll 1$ is disregarded, the attenuation can be approximated:

$$Att = 20\log_{10}\left|1-k^2\frac{Vl_0}{s}+\frac{l_0}{l_i}\right| \quad \mathrm{dB} \qquad \cdots\cdots\cdots(43)$$

which in the extreme low frequency range becomes $20\log_{10}|1+(l_0/l_i)_{01}|$ dB, or, in a more exact form of expression, $20\log_{10}|1+(l_0/li)+(s/S)(l/l_i)|$ dB.

4. Calculation and experimentation

In the foregoing sections, a theoretical analysis has been presented for sound sources having the characteristics of constant velocity ($U_1'=U_1$) and constant pressure ($p_1'=p_1$). In this section, numeral combutation using these equations will be compared to experiments.

4.1 Constant velocity sound source

The sound originating from a high-pressure cylinder may be, in theory, very close

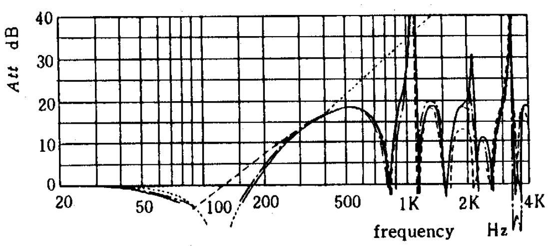

Fig. 3 Computed Att. [I. Constant velocity sound source. The solid line is Eq. (11), the chain line is Eq. (15), the broken line is Eq. (20) and the dotted line is Eq. (39)]

to the constant velocity sound source. This particular type of sound source involves so many non-linear elements that it is exceedingly difficult to form a perfect constant velocity sound source.

Figure 3 shows calculation of the frequency characteristics of the attenuation for a single-stage cavity muffler connected with a constant velocity sound source. Dimensions of the inlet pipe are $l_i = l' = 40$ mm (ϕ22 mm), cavity $L = 200$ mm (ϕ58 mm), outlet pipe $l_0 = 150$ mm (ϕ22 mm): $l_{i1} = 0$ and the end of l_0 is inserted into the cavity by $l_{01} =$ 80 mm. In this figure, the solid line denotes the calculated value using Eq. (11), the chain line the computed results using Eq. (15), the broken line the results using Eq. (20), and the dotted line the results using Eq. (39).

Computed results on the basis of practical Eq. (20) are almost consistent with those depicted by the full line Eq. (11) within the frequency range exceeding 300 Hz, and the results using the low-frequency equation Eq. (39) are with the frequency range below 300 Hz. In practice, it will be convenient to make compution using a combination of the two equations; although in calculations using the approximate Eq. (20), errors will be pronounced within the area where $\tan kl_i$ approaches infinity. In this case,

$$kl_i = \frac{2n-1}{2}\pi \ (n\text{: positive integer}) \qquad \cdots\cdots\cdots(44)$$

and, hence, the corresponding frequency is:

$$f_i = \frac{(2n-1)c}{4l_i} = 2100,\ 6300, \cdots \text{ Hz} \qquad \cdots\cdots\cdots(45)$$

This necessitates taking a certain amount of care in the vicinity of these frequencies. The resonance frequency of the system is 140 Hz.

Resonance is also produced written the cavity length L, and the corresponding frequency is:

$$f_L = \frac{nc}{2L} = \frac{340n}{2\times 0.2} = 850,\ 1700,\ 2550, \cdots \text{ Hz} \qquad \cdots\cdots\cdots(46)$$

while along the outlet (tail) pipe of length l_0 the frequency is

$$f_0 = \frac{nc}{2l_0} = \frac{340n}{2\times 0.15} = 1130,\ 2260, \cdots\ Hz \qquad \cdots\cdots\cdots(47)$$

No attenuation effect is produced at these frequencies.

4.2 Constant pressure sound source

An example of the constant pressure sound source may be the sound which escapes from an opening in a vessel adequately covered with sound absorption material.

Figure 4 is an example of frequency characteristics of the calculated attenuation for a constant pressure sound source using a muffler with the same dimensions as those in Fig. 3. In the figure, the full line denotes the calculated results using Eq. (27), the chain line the computed results using Eq. (30), the broken line the results using Eq. (33)

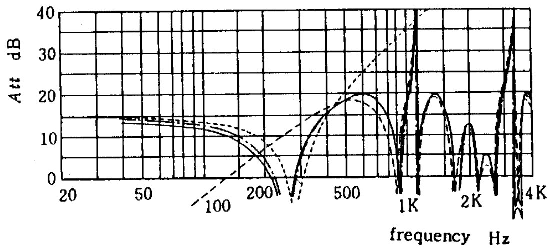

Fig. 4 Computed *Att.* [II. Constant pressure sound source. The solid line is Eq. (27), the chain line is Eq. (30), the broken line is Eq. (33) and the dotted line is Eq. (42)]

and the dotted line the results using Eq. (42). In this case, the calculations using the approximate Eq. (33) in the range above 400 Hz and those calculations using the low frequency Eq. (42) in the range below 400 Hz, are comparatively close to those shown by the full line Eq. (27). This result serves to verify that both expressions are of good approximations, but Eq. (33) is inconsistent for calculations in the area where $\cot kl_i$ approaches infinity. Here,

$$kl_i = n\pi \; (n: \text{ positive integer}) \qquad \cdots\cdots\cdots(48)$$

$$\therefore \quad f_i = \frac{nc}{2l_i} = 4200,\ 8400, \cdots \text{ Hz} \qquad \cdots\cdots\cdots(49)$$

so ample care is necessary within the vecinity of these frequencies. Also, no attenuation effect is produced at the frequencies f_L, f_0, etc., as made evident by Eqs. (41) and (42), nor at the resonance frequency (260 Hz) of the system.

Figure 5 shows a comparison between the observed results of a pure tone experiment using a constant pressure sound source device of Brüel and Kjær Co., and the results of calculations using Eq. (30).

Muffler dimensions are set as

ϕ22 mm × 150 mm ($l' = l_i$)	Inlet pipe
ϕ53 mm × 200 mm	Cacity
ϕ22 mm × 150 mm	Outlet pipe

and the inserted length (l_{i1}) of the inlet pipe is set at 50 mm; while that (l_{01}) of the outlet pipe varies from 20, 40, 50, 60, 80, 100, to 130 mm. Also for the outlet pipe l_0, of course, the end-correction at both ends is necessary. In case the pipe is open to free space, the correction value at one end is assumed to be:

$$\Delta l = 0.6a \; (a: \text{ radius}) \qquad \cdots\cdots\cdots(50)$$

and, therefore, an outlet pipe $2\Delta l$ shorter, i.e., approximately 13 mm shorter, was used in the experiment.

The upper limet frequency is 4 kHz, because higher frequencies producing resonance perpendicular to the axis of the cavity are beyond the scope of this article, which

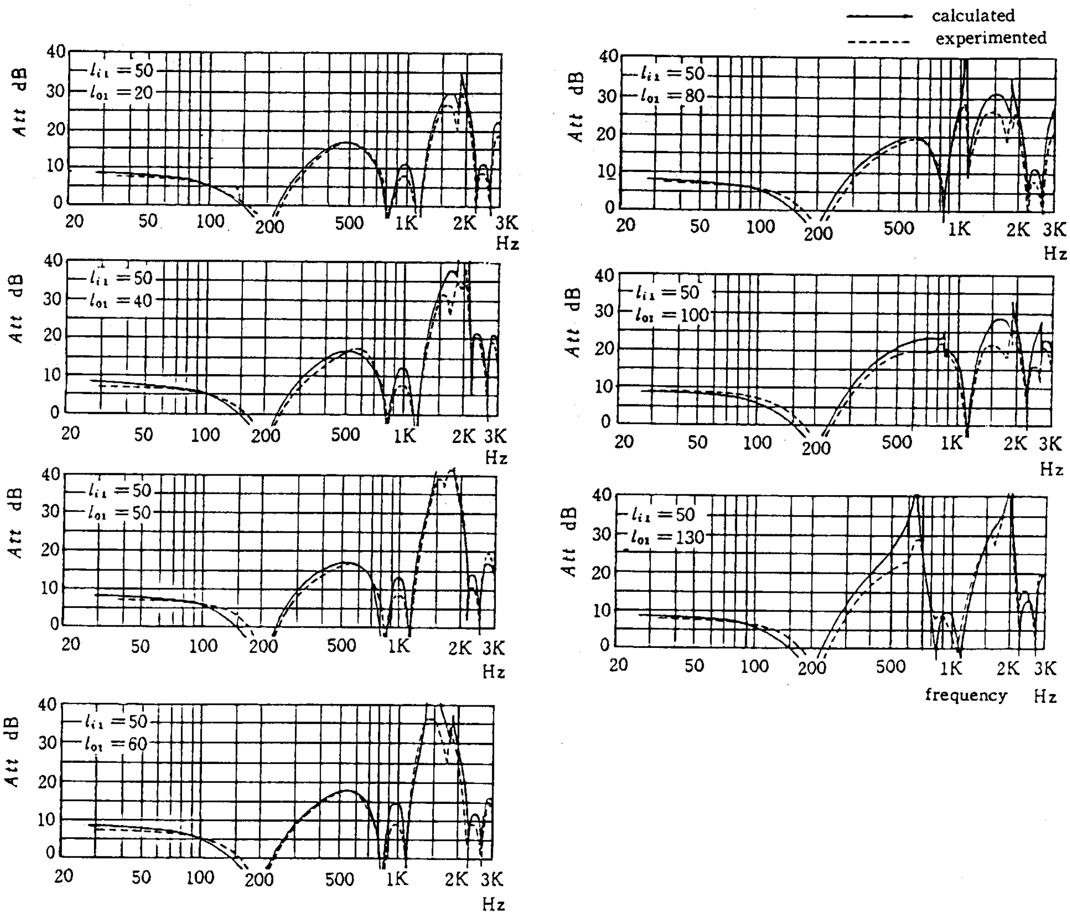

Fig. 5 Comparison between the results of computation using Eq. (30) and pure tone experiments.

assumes plane wave conditions. The frequency f_c is:

$$f_c = 1.22\frac{c}{2a} \fallingdotseq 4 \text{ kHz} \qquad \cdots\cdots\cdots(51)$$

On the other hand, substituting the outlet radius $a = 11$ mm in

$$ka < 0.6 \qquad \cdots\cdots\cdots(52)$$

will give this frequency

$$f < \frac{0.6c}{2\pi a} \fallingdotseq 3 \text{ kHz} \qquad \cdots\cdots\cdots(53)$$

Because of this, the upper limit frequency was set at 3 kHz in this paper for the calculations as well as for the experiment. When comparing the calculated (solid line) with the experimental (broken line) results, we see considerable agreement.

For example, when $l_{i1} = l_{01} = 50$ mm considerable attenuation occurs at the following frequency (1/4–1/4 effect):

$$f = \frac{c}{L} = 1700 \text{ Hz} \qquad \cdots\cdots\cdots(54)$$

When $l_{i1}=50$ mm and $l_{01}=100$ mm

$$f=\frac{nc}{2L}=850,\ 1700,\ 2550,\cdots\ \text{Hz} \qquad \cdots\cdots\cdots(55)$$

within which resonance decays (1/4–1/2 effect). In either case, nontheless, resonance at the outlet pipe is produced at the following frequency.

$$f_0=\frac{nc}{2l_0}=1130,\ 2260,\cdots\ \text{Hz} \qquad \cdots\cdots\cdots(56)$$

5. Conclusions

Summing up the points mentioned in this paper, it is concluded as follows:

(1) This paper presents precise equations for the attenuation of single stage reactive mufflers designed to reduce the noise radiating into free space.

(2) The precise equation for the attenuation of mufflers connected to a constant velocity sound source is given by Eq. (11) or Eq. (14).

(3) The precise equation for the attenuation of mufflers connected to a constant pressure sound source is given by Eq. (27) or Eq. (29).

(4) A general expression of the attenuation for a low frequency sound source is given by Eq. (36).

(5) The calculated values have been found in good agreement with those obtained from the experiments.

(6) The effects of flow were neglected in this paper.

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