Calculations of Critical Current Densities for Type-II Superconductors (I)

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Abstract

In this paper, for mixed state in Type-II superconductors London equation is solved by using normal core model which gives boundary conditions in the same way as Coffey, to obtain the interaction force between a fluxoid and fluxoids lattice. This interactien force that is repulsive is considered pinning force as Coffey has proposed.

As loss mechanism normal eddy-current damping and cycloid damping are considered and we get the expressions of viscous coefficient and induced voltage across superconductor, that are rather different from the ones given by Kim, *et al.* Our expression of voltage across the sample does not equal to $v_L B/c$, which is usually adopted in flux flow theory, where v_L is fluxoid velocity and B magnetic induction.

Two forms of critical current density as a function of reduced external magnetic field for a given criterion voltage appeared in sample are found and plotted for various values of parameters. Also discussion is made about voltage induced by fluxoid motion in superconductor.

1. INTRODUCTION

As is well known, non-ideal Type-II superconductors show resistance in the mixed state with transverse magnetic field if transport current is injected above some critical value, and this means voltage appeares across the superconductors. The induced longitudinal voltage is explained to be caused by flux-flow phenomenon which means the one that quantized fluxes, which is also called fluxoids, in the superconductors are driven to move by the Lorentz force JB_c against pinning force. This phenomenon is well discussed by Kim *et al.*[1] on the phenomenological sides.

Critical current density vs. external magnetic field strength curves, namely J_c-H curves, are fundamental materials when Type-II superconductors are used for many applications, especially power machines because the loss in the superconductors has great importance there. We show a typical J_c-H curve in Fig. 1 .[2]

Although there are some interpretations[3)-7)] of observed J_c-H curves which have empirical expressions with adjusting parameters, few papers have reported grounded on rather pure theories.

Coffey[8] has calculated critical current density J_c by solving the London equation for simple normal core medel. His expression gives good agreement with experimental data for some samples but because of its uniqueness except multipling factor many samples remain outside the expression. In Fig. 2 a J_c-H curve calculated by Coffey's expression is shown.

In this paper, we try to find new equations to fit experimental data by reforming Coffey's theory and give some discussions about loss mechanism in superconductors.

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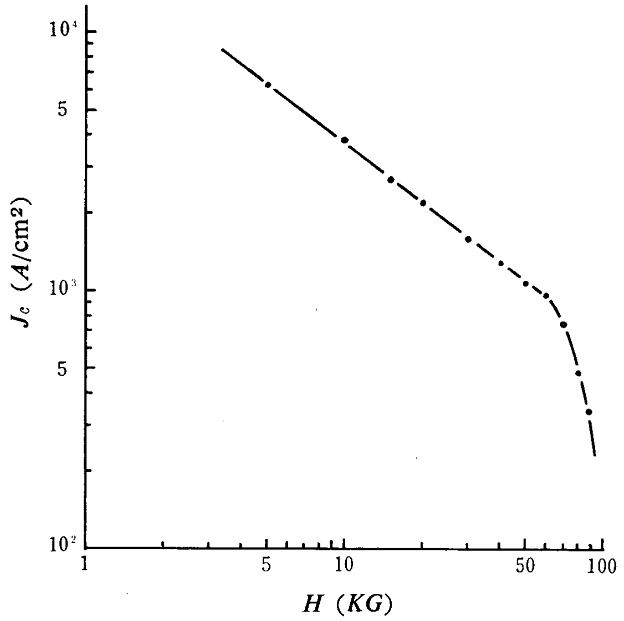


Fig. 1 One of the typical shapes of critical current density vs. applied transverse magnetic field. Sample is Nb 50%−Zr alloy that is approximately 7/8−in long rectangular rod, and criterion voltage is about 10^{-8} V

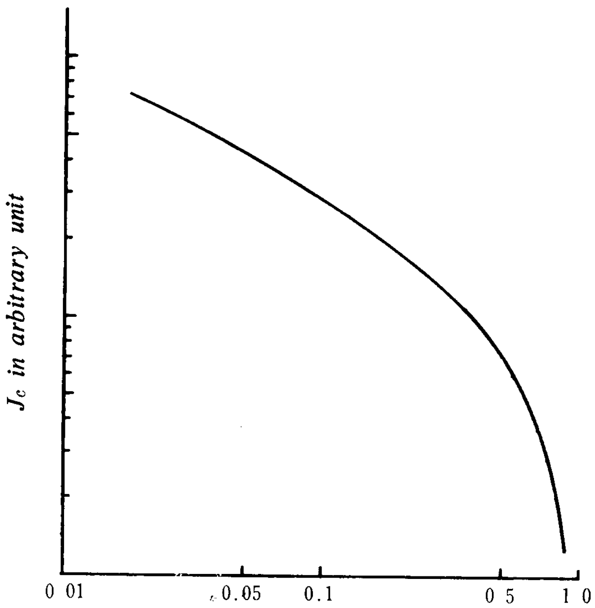


Fig. 2 Calculated curve by Coffey's[3] expression

2. MODIFIED LONDON MODEL

The theory described in this section is almost the same by Coffey[3] except details.

London equation[5] may be represented as

$$\mathbf{h}+\lambda^2 \text{curl curl } \mathbf{h}=0 \tag{1}$$

for $\lambda \gg \xi$, where **h** is the local field in the superconductor, λ London's penetration depth, and ξ coherence length.

De Gennes *et al*[10]. replace the right hand side of Eq. (1) by δ-function, that is,

$$\mathbf{h}(\mathbf{r})+\lambda^2 \text{ curl curl } \mathbf{h}(\mathbf{r})=N\phi_0\, \delta(\mathbf{r}). \tag{2}$$

$$(\lambda \gg \xi)$$

Using Maxwell's equation

$$\text{curl } \mathbf{h}=\frac{4\pi}{c}\mathbf{j}, \tag{3}$$

Eq. (2) leads to the following equation ;

$$\iint \mathbf{h}\, d\mathbf{s}+\frac{4\pi\lambda^2}{c}\oint_R \mathbf{j}\, d\mathbf{l}=N\phi_0 \tag{4}$$

which is called the London fluxoid quantization condition, where ϕ_0 is one quantized flux, i. e. $\phi_0=hc/2e=2.07\times10^{-7}$ Gcm^2. N should be taken as an integer but there are some experiments[11] which imply that N depends on samples and more over show there exist defects such as flux line dislocations, point defects, holes and so on in the fluxoid lattice. Thus we had better consider N is not always an integer but a positive value in the average sense. Except origin Eq. (2) becomes

$$\nabla^2 \mathbf{h}(\mathbf{r})=\mathbf{h}(\mathbf{r})/\lambda^2, \tag{5}$$

which gives the expression of $\mathbf{h}(\mathbf{r})$. If we choose a cylindrical coordinates in an infinite slab superconductor, as shown in Fig. 3, the general solution of Eq. (5) may be written as the

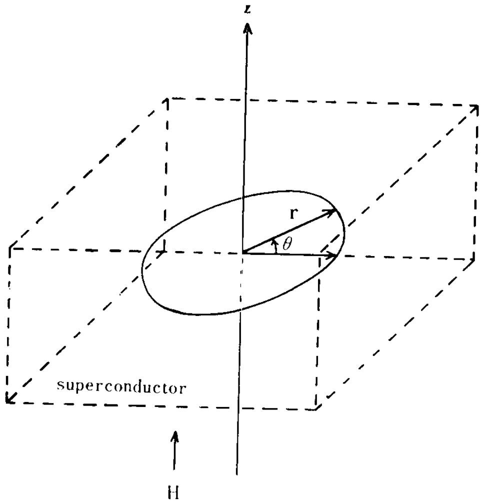


Fig. 3 Coordinate system in slab superconductor with perpendicular magnetic field H for its surface

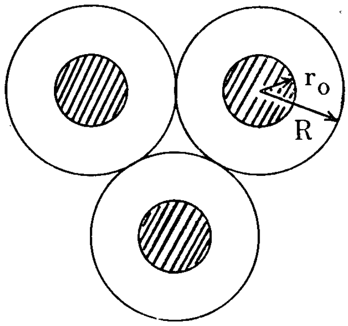


Fig. 4 (a) Simple normal core model. The region with oblique lines is normal and other region is superconducting

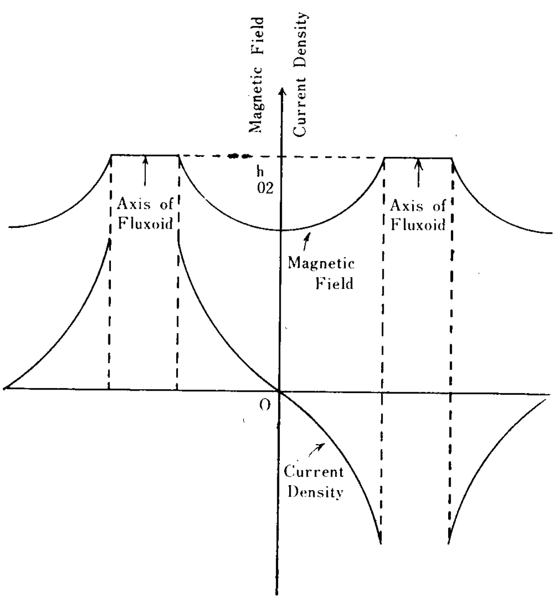


Fig. 4 (b) Distributions of magnetic field and current density among fluxoids are shown schematically

following form :

$$h(r/\lambda) = CI_0(r/\lambda) + DK_0(r/\lambda), \qquad (6)$$

where I_0 and K_0 are modified Bessel functions, and both C and D are constant. The current density can be obtained from Eqs. (3) and (6) :

$$j(r/\lambda) = -\frac{c}{4\pi\lambda}\left[CI_1(r/\lambda) - DK_1(r/\lambda)\right] . \qquad (7)$$

As the K-functions are infinite at the origin of coordinates, we cut off the solutions at some radius r_0, inside region of which is normal with no current flow, and outside region is superconducting with supercurrent. At the boundary of $r=r_0$ some critical current density flows. This core model is shown in Fig. 4 (a)[3]. Magnetic field and current density distributions inside and outside a core are schematically represented in Fig. 4 (b)[3]. On the basis of this core model we can determine the constant C and D in Eqs. (6) and (7).

2.1 Distributions in an Isolated Fluxoid

In the case that one fluxoid exists across infinitely spacious slab we impose the following boundary conditions on Eqs. (6) and (7) :

$$h(r_{01}/\lambda) = h_{01} = \text{constant}$$

$$\left[h(r/\lambda)\right]_{r\to\infty} = 0 . \qquad (8)$$

Then magnetic field and current density are obtained as follows :

$$h_1(r/\lambda) = h_{01}\frac{K_0(r/\lambda)}{K_0(r_{01}/\lambda)} \qquad (9)$$

$$j_1(r/\lambda) = \frac{ch_{01}}{4\pi\lambda}\frac{K_1(r/\lambda)}{K_0(r_{01}/\lambda)} \qquad (10)$$

for $r \geqq r_{01}$.

Putting these formulas into quantization condition, Eq. (4), we find

$$\pi r_{01}^2 h_{01} + 2\pi\lambda h_{01} r_{01} \frac{K_1(r_{01}/\lambda)}{K_0(r_{01}/\lambda)} = N\phi_0 \tag{11}$$

or

$$\pi r_{01}^2 h_{01} + \frac{8\pi^2\lambda^2 r_{01}}{c} j_1(r_{01}/\lambda) = N\phi_0 \tag{12}$$

with quantized flux $N\phi_0$. The first term is negligible compared with the second for large κ', which is defined by

$$\kappa' = \frac{\lambda}{r_{01}} \tag{13}$$

and then

$$2h_{01} \frac{\lambda}{r_{01}} \frac{K_1(r_{01}/\lambda)}{K_0(r_{01}/\lambda)} = H_{c2} \tag{14}$$

where

$$\frac{N\phi_0}{\pi r_{01}^2} = H_{c2} \tag{15}$$

is applied.

2.2 Distributions in Latticed Fluxoids

In arbitrary magnetic flux density B we consider that flux of $N\phi_0$ is contained within radius R, i. e.

$$B = \frac{N\phi_0}{\pi R^2} \quad . \tag{16}$$

We regard N as independent of B but dependent on samples.

As shown in Fig. 4 (a), fluxoids are arranged to make triangular lattice[12] so that free energy may become minimum.

In this case we choose

$$\begin{aligned} &h_2(r_{02}/\lambda) = h_{02} \\ &j_2(R/\lambda) = 0 \\ &\qquad (0 < r_{02} \leqq R) \end{aligned} \tag{17}$$

as boundary conditions. h_{02} is constant for distance r but dependent on magnetic flux density B or radius R. Although we regard $r_{02} = r_{01}$ later, we distinguish between r_{02} and r_{01} for easy understanding at present. From Eqs.(6), (7) and (17),

$$h_2(r/\lambda) = h_{02} \left\{ \frac{K_1(R/\lambda)I_0(r/\lambda) + K_0(r/\lambda)I_1(R/\lambda)}{K_1(R/\lambda)I_0(r_{02}/\lambda) + K_0(r_{02}/\lambda)I_1(R/\lambda)} \right\} \tag{18}$$

$$j_2(r/\lambda) = \frac{ch_{02}}{4\pi\lambda} \left\{ \frac{K_1(r/\lambda)I_1(R/\lambda) - K_1(R/\lambda)I_1(r/\lambda)}{K_1(R/\lambda)I_0(r_{02}/\lambda) + K_0(r_{02}/\lambda)I_1(R/\lambda)} \right\} \tag{19}$$

are obtained for $r_{02} \leqq r \leqq R$. We can see that current density $j_2(r/\lambda) \geqq 0$ for $0 < r \leqq R$ because K-functions decrease monotonously while I-functions monotonously increase[13]. Putting these equations into quantization condition Eq. (4), we get

$$\pi r_{02}^2 h_{02} + 2\pi\lambda h_{02} r_{02} \left\{ \frac{K_1(r_{02}/\lambda)I_1(R/\lambda) - K_1(R/\lambda)I_1(r_{02}/\lambda)}{K_1(R/\lambda)I_0(r_{02}/\lambda) + K_0(r_{02}/\lambda)I_1(R/\lambda)} \right\} = N\phi_0 \tag{20}$$

or

$$\pi r_{02}^2 h_{02} + \frac{8\pi^2\lambda^2 r_{02}}{c} j_2(r_{02}/\lambda) = N\phi_0 \tag{21}$$

or

$$h_{02} + \frac{8\pi\lambda^2}{c r_{02}} j_2(r_{02}/\lambda) = H_{c2} \quad . \tag{22}$$

Apparently, Eqs. (9), (10), (18) and (19) seem to be different from ones obtained by Coffey[8] but the both are exactly same being considered Eqs. (11) and (20).

It may be noticed that when radius R go to infinite Eqs. (18) and (19) become the distributions in the case of an isolated fluxoid, Eqs. (9) and (10).

The field strength in the normal core, which is represented by h_{02}, may be led from Eq. (18) for $r_{02}/\lambda \ll 1$ and $R/\lambda \ll 1$, that is,

$$h_{02}=B \tag{23}$$

where we have used the following approximate expressions upon modified Bessel functions :

$$K_0(z)\approx ln(1/z),\quad K_1(z)\approx 1/z,$$
$$I_0(z)\approx 1,\qquad I_1(z)\approx z/2, \tag{24}$$

when $0<z\ll 1$.

3. FORCE OF FLUXOID LATTICE ON A FLUXOID

A fluxoid symmetrically surrounded by other fluxoids will experience no force but if shifted from its equilibrium position it should be forced back to the original position. We consider here such situation that a fluxoid of $N\phi_0$ is displaced to a position where current density j_3, which is produced by fluxoid lattice, flows. In this case force of $N\phi_0 j_3/c$ acts the fluxoid towards the original point where it was situated initially, and we may write pinning force as

$$f_p=N\phi_0 j_3/c. \tag{25}$$

By Coffey current density j_3 contributed by fluxoid lattice is approximately given by

$$j_3=j_2-j_1. \tag{26}$$

If we regard $r_{01}=r_{02}=r_0$ in the same way as Coffey, we can get

$$j_3=-\left(\frac{ch_{01}}{4\pi\lambda}\right)\frac{K_1(R/\lambda)}{K_0(r_0/\lambda)}\left(\frac{I_1(r/\lambda)K_0(r_0/\lambda)+I_0(r_0/\lambda)K_1(r/\lambda)+\frac{2\lambda}{r_0}(K_1(r_0/\lambda)I_1(r/\lambda)-K_1(r/\lambda)I_1(r_0/\lambda))}{K_1(R/\lambda)I_0(r_0/\lambda)+K_0(r_0/\lambda)I_1(R/\lambda)+\frac{2\lambda}{r_0}(K_1(r_0/\lambda)I_1(R/\lambda)-K_1(R/\lambda)I_1(r_0/\lambda))}\right). \tag{27}$$

When $r_0/\lambda \ll 1$, $r/\lambda \ll 1$ and $R/\lambda \ll 1$, we can use approximate equations, Eq. (24), for modified Bessel functions, to obtain simpler expression, that is,

$$j_3=-\frac{ch_{01}}{4\pi ln(\lambda/r_0)}\left(\frac{r}{R^2}\right)\ . \tag{28}$$

Here r should be replaced by displacement Δr. Although this equation should be applied for the region of $r_0\leqq r\leqq R$, we use it for $0<r<R$ in approximation.

It should be noticed that we have considered $h_{01}\neq h_{02}$ to derive j_3, i. e. Eq. (27), or (28) nevertheless Coffey[8] has not done so. Then Eq. (28) gives $j_3=0$ for $r=0$, that is equiliblium position, but Coffey's equation formally gives infiniay for $r=0$.

4. INDUCED VOLTAGE

When fluxoids, number of n_f per unit area, move stationarily with velocity v_L across slab-shaped superconductor, electric field E_f will be generated, i. e.

$$E_f=-\frac{1}{c}\frac{d\Phi}{dt}=-\frac{1}{c}v_L n_f(N\phi_0) \tag{29}$$

which is proposed by Kim and co-workers[1]. And more over additional field E_i may exist within normal core :

$$E_i=-\frac{\hbar v_L}{2er_0^2} \tag{30}$$

that is insisted by Stephen and Bardeen[14], where e is electron charge, r_0 normal core radius and $\hbar$ Planck's constant divided by 2π. Thus total field along the superconductor may be written by

$$E_t = E_f + E_i$$
$$= -\left(\frac{\pi r_0^2 n_f H_{c2}}{c} + \frac{\hbar}{2 e r_0^2}\right) v_L \quad . \tag{31}$$

Denoting the appeared voltage per unit length along the superconductor by E, measuring transport current density by J, density of fluxoid core by n_c and normal conductivity of the core by σ, the loss per unit volume in the sample

$$P = EJ \tag{32}$$

should be equal to consumption in normal core, that is,

$$P = n_c(\pi r_0^2 \sigma) E_t^2 \cdot \tag{33}$$

The number of flowing fluxoids, n_f, is the one of unpinned fluxoids, since we consider pinned fluxoids, which is denoted by n_p per unit area, cannot flow, and

$$n_f = n_c - n_p, \tag{34}$$

with

$$n_c = \frac{B}{N\phi_0} \quad , \tag{35}$$

$$n_p = \frac{B_p}{N\phi_0} \tag{36}$$

and

$$n_f = \frac{1}{\pi r_0^2}\left(\frac{B}{H_{c2}} - \frac{B_p}{H_{c2}}\right) . \tag{37}$$

We are restricted here to the case of $n_c > n_p$ ie. .

$$B > B_p . \tag{38}$$

With these expressions we can rewrite Eq. (33) :

$$P = \sigma \frac{H_{c2}^2}{c^2}\left(\frac{B}{H_{c2}}\right)\left(\frac{B}{H_{c2}} - \frac{B_p}{H_{c2}} + \frac{1}{2N}\right)^2 v_L^2 . \tag{39}$$

On the other hand loss P may be expressed with the viscosity coefficient η of the medium as follows :

$$P = n_f \eta v_L^2 . \tag{40}$$

Fluxoids begin to flow when Lorentz force F_L per unit volume overcomes pinning force F_p per unit volume, and force balance equation may be expressed as

$$F_L - F_p = n_f \eta v_L . \tag{41}$$

From Eqs. (39), (40) and (41), we find

$$v_L = \frac{J(B/H_{c2}) - (c/H_{c2})F_p}{\sigma(H_{c2}/c)(B/H_{c2})(B/H_{c2} - B_p/H_{c2} + 1/2N)^2} \tag{42}$$

and

$$\eta = \pi r_0^2 \sigma \left(\frac{H_{c2}}{c}\right)^2 \frac{(B/H_{c2})(B/H_{c2} - B_p/H_{c2} + 1/2N)^2}{B/H_{c2} - B_p/H_{c2}}$$
$$= \frac{(N\phi_0)\sigma H_{c2}}{c^2} \frac{(B/H_{c2})(B/H_{c2} - B_p/H_{c2} + 1/2N)^2}{B/H_{c2} - B_p/H_{c2}} . \tag{43}$$

When $B >> B_p$ and $N = 1$ Eq. (43) becomes

$$\eta = \frac{\phi_0 \sigma H_{c2}}{c^2}\left(\frac{B}{H_{c2}} + \frac{1}{2}\right)^2 \tag{44}$$

and this formula is the same one as Kim *et al.*[1] have shown except the term of 1/2. Although Kim's expression is too small to account for the observation at low field, our equation Eq. (43) seems to be agreeable because the denominator becomes small at low field.

Induced voltage along the sample per unit length becomes

$$E=\frac{\{J(B/H_{c2})-(c/H_{c2})F_p\}^2}{\sigma J(B/H_{c2})(B/H_{c2}-B_p/H_{c2}+1/2N)^2} \tag{45}$$

in the presence of current flow J.

5. CRITICAL CURRENT DENSITY

We can now calculate the critical current density J_c, which is defined in such a way as some criterion voltage E_c per unit length along the sample appears. Then Eq. (45) leads

$$J_c=\left(\frac{\sigma E_c}{2}\right)\Big[\left(\frac{2}{\sigma E_c}\right)\left(\frac{cF_p}{H_{c2}}\right)\frac{1}{x}+\frac{1}{x}(x-B_p/H_{c2}+1/2N)^2 \pm\sqrt{\left\{\left(\frac{2}{\sigma E_c}\right)\left(\frac{cF_p}{H_{c2}}\right)\frac{1}{x}+\frac{1}{x}(x-B_p/H_{c2}+1/2N)^2\right\}^2-\left(\frac{2}{\sigma E_c}\right)^2\left(\frac{cF_p}{H_{c2}}\right)^2\frac{1}{x^2}}\Big] \tag{46}$$

where we have put

$$x=\frac{B}{H_{c2}}. \tag{47}$$

For a sample with large $\kappa'=\lambda/r_c$, magnetic flux density B can be replaced by external magnetic field H with little error for its magnetization is very small. We had better to adopt the lower sign in above expression because if pinning force does not exist critical current density should vanish. It will be instructive to mention that if criterion voltage E_c is negligibly small, Eq. (46) reduces

$$J_c=\frac{cF_p}{B}, \tag{48}$$

which agrees with usual expression used in flux flow theory, for instance, in the pape of Friedel *et al.*[15] or Irie *et al.*[7].

As we consider the case of $B>B_p$, the pinning force density F_p may be given by

$$F_p=n_rf_p, \tag{49}$$

where f_p is local pinning force. Using Eqs. (27), (28) and (49), we get

$$F_p=\beta\gamma\left(\frac{H_{c2}}{c}\right)\left(\frac{r_0\Delta r}{R^2}\right), \tag{50}$$

where

$$\beta=\frac{ch_{01}}{4\pi r_0\ln(\lambda/r_0)}=\frac{cH_c}{4\pi\lambda} \tag{51}$$

and

$$\gamma=\frac{B_p}{H_{c2}} \tag{52}$$

and also, H_c is determined so that condensation energy should become $H_c^2/8\pi$. By putting

$$\alpha=\frac{\sigma E_c}{2} \tag{53}$$

and substituting Eq. (50) into Eq. (46), we find

$$J_c=\alpha\Big[\frac{\beta\gamma}{\alpha}\left(\frac{r_0\Delta r}{R^2}\right)\frac{1}{x}+\frac{1}{x}\left(x-\gamma+\frac{1}{2N}\right)^2-\sqrt{\left\{\frac{\beta\gamma}{\alpha}\left(\frac{r_0\Delta r}{R^2}\right)\frac{1}{x}+\frac{1}{x}\left(x-\gamma+\frac{1}{2N}\right)^2\right\}^2-\left\{\frac{\beta\gamma}{\alpha}\left(\frac{r_0\Delta r}{R^2}\right)\frac{1}{x}\right\}^2}\Big] \tag{54}$$

for $x>\gamma$ or

$$J_c=\alpha\Big[k\left(\frac{r_0\Delta r}{R^2}\right)\frac{1}{x}+\frac{1}{x}(x+nn)^2-\sqrt{\left\{k\left(\frac{r_0\Delta r}{R^2}\right)\frac{1}{x}+\frac{1}{x}(x+nn)^2\right\}^2-\left\{k\left(\frac{r_0\Delta r}{R^2}\right)\frac{1}{x}\right\}^2}\Big] \tag{55}$$

$$(x>\gamma)$$

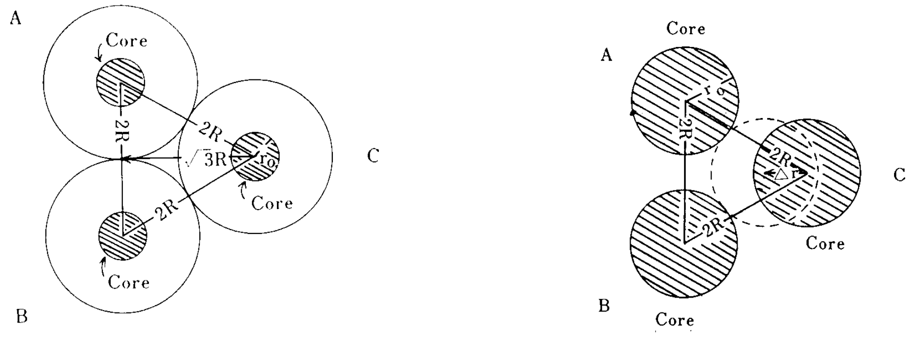
where

$$k=\frac{\beta\gamma}{\alpha} \tag{56}$$

$$nn=-\gamma+\frac{1}{2N} \tag{57}$$

Now, we must choose displacement Δr in appropriate manner. We try to show some cases next.

〔I〕 When magnetic flux density B is small and the distance R between fluxoids is large as shown in Fig. 5 (a), fluxoid C which is driven by Lorentz force will pass through the middle of two fluxoids owing to repulsive force among them. This is indicated by an arrow. In this path fluxoid C will experience maximum repulsive force on the line AB. So that for the calculation of pinning force we take



(*a*) *When 2ro ≦ R.* (*b*) *When 2ro>R.*

Fig. 5 Configurations of fluxoids for the choice of displacement △r

$$\Delta r=\sqrt{3}\,R \tag{58}$$

for $2r_0 \leqq R$.

On the other hand, when B is large and $2r_0 > R$, fluxoid C will knock against other fluxoid A and B in the movement. Then we put

$$\Delta r=\sqrt{3}\,R-\sqrt{4r_0^2-R^2} \tag{59}$$

for $2r_0 > R$ to get maximum resistive force (Fig. 5 (b)).

In this choice of Δr Eq. (55) becomes

$$J_c=\alpha\left[k\frac{\sqrt{3}}{\sqrt{x}}+\frac{1}{x}(x+nn)^2-\sqrt{\left\{k\frac{\sqrt{3}}{\sqrt{x}}+\frac{1}{x}(x+nn)^2\right\}^2-\left\{k\frac{\sqrt{3}}{\sqrt{x}}\right\}^2}\,\right] \tag{60}$$

$$(0<x\leqq 0.25)$$

and

$$J_c=\alpha\left[k\frac{\sqrt{3}-\sqrt{4x-1}}{\sqrt{x}}+\frac{1}{x}(x+nn)^2-\sqrt{\left\{k\frac{\sqrt{3}-\sqrt{4x-1}}{\sqrt{x}}+\frac{1}{x}(x+nn)^2\right\}^2-\left\{k\frac{\sqrt{3}-\sqrt{4x-1}}{\sqrt{x}}\right\}^2}\,\right] \tag{61}$$

$$(0.25\leqq x\leqq 1)$$

This curves as a function of x are shown in Fig. 6 where $nn=0.5$ and k has several values, and Fig. 7 where $k=10^{-1}$ and nn varies.

〔II〕 If we choose such that

$$\Delta r=R-r_0\ , \tag{62}$$

critical current density may be obtained as follows :

$$J_c=\alpha\left[k\left(\frac{1}{\sqrt{x}}-1\right)+\frac{1}{x}(x+nn)^2-\sqrt{\left\{k\left(\frac{1}{\sqrt{x}}-1\right)+\frac{1}{x}(x+nn)^2\right\}^2-\left\{k\left(\frac{1}{\sqrt{x}}-1\right)\right\}^2}\,\right] \tag{63}$$

This results are shown in Fig. 8 for constant nn and various k's, and in Fig. 9 for constant k and various nn's.

6. CONCLUTIONS AND DISCUSSION

We have derived the relation between induced voltage and transport current in Type-II superconductors with transverse magnetic field through the consideration of appropriate loss mechanism.

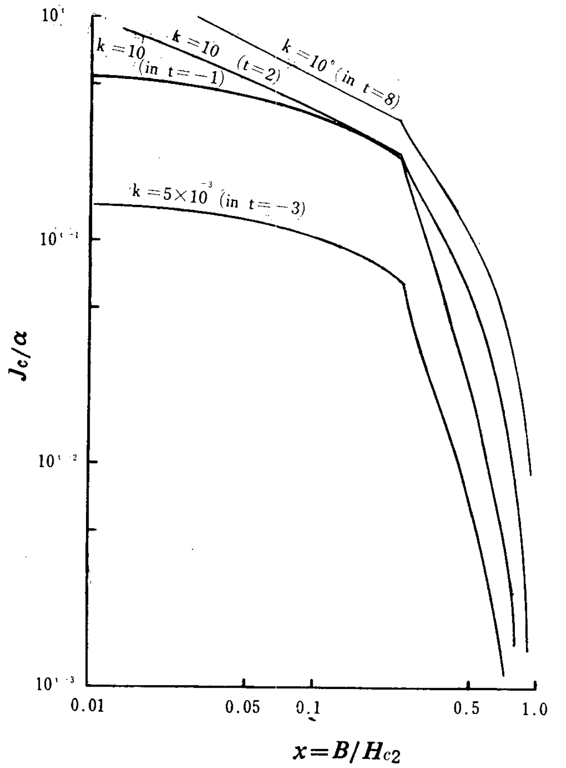


Fig. 6 Logarithmic plot of reduced critical current density as calculated using Eqs. (60) and (61) for nn=0.5 and various k values as a function of normalized magnetic field

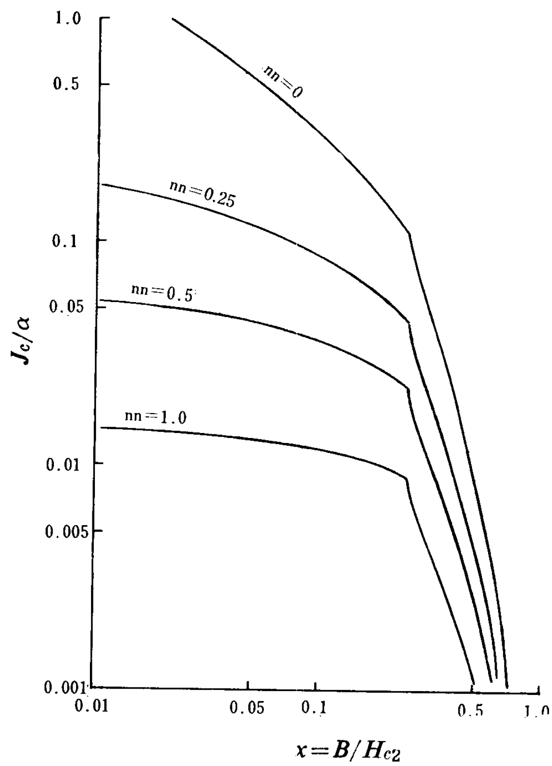


Fig. 7 Logarithmic plot of reduced critical current density as calculated using Eqs. (60) and (61) for $k=10^{-1}$ and various nn values as a function of normalized magnetic field

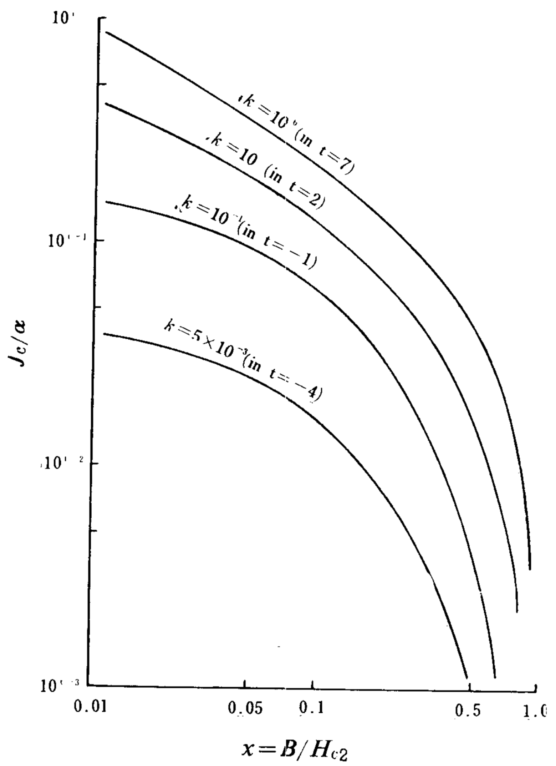


Fig. 8 Logarithmic plot of reduced critical current density as calculated using Eq. (63) for nn=0.5 and various k values as a function of normalized magnetic field

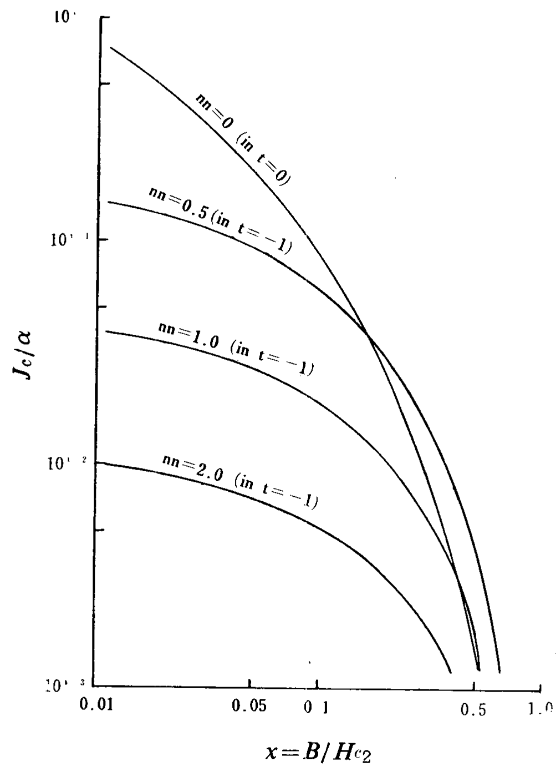


Fig. 9 Lagarithmic plot of reduced critical current density as calculated using Eq. (63) for $k=10^{-1}$ and various nn values as a function of normalized magnetic field

To get pinning force, we have calculated the repulsive force which acts between a fluxoid and other fluxoids by solving the London equation for a normal core model. Using obtained pinning force we have found critical current density representation Eqs. (60), (61) and (63) as a function of external magnetic field for a given criterion voltage, which is shown in from Fig. 6 to Fig. 9.

As these curves do not fall down sharply or in very small interval near H_{c2}, it seems to be difficult for our expressions to agree well with experimental data except some ones including the region of large x, and further investigations should be done.

There still remain some questionable or remarkable points in our theory and we would like to describe them below.

1) As usually done, we have written down Eq. (32), which means that loss P dissipated by conductor with unit length and unit cross-sectional area is given by the product of voltage E and current J in it, but does it hold always? Now we consider a simple electric circuit like Fig. 10, where voltage source S with voltage V_s, resistor R with resistance R_r and D. C. generator G, which is assumed to be constructed by perfect conductor without resistance, are connected in series. When generator is stopped, current flowing in the circuit is given by

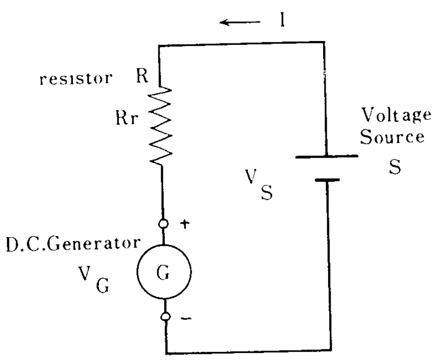


Fig. 10 Circuit with D. C. Generator which is made of perfect conductor

$$I_1 = \frac{V_s}{R_r} \quad . \tag{64}$$

Next, generator is rotated and produces voltage of V_G against V_S, then current

$$I_2 = \frac{V_S - V_G}{R_r} = I_1 - \frac{V_G}{R_r} \tag{65}$$

will flow through the generator. Though generator has voltage V_G and current I_2, it will not dissipate the power $V_G \times I_2$, because the generator is made of perfect conductor. The work made for rotating the generator is restored to the source S. From this fact we can understand that electromotive force induced by magnetic induction does not result in loss. Thus this kind of power should be removed in our concerns.

2) In our theory we have considered that a pin site catches a fluxoid more strongly than fluxoids lattice, so that if fluxoid number is larger than pin site one, that is, $B > B_p$ critical current density may be decided by rigidity of fluxoids lattice and if fluxoid number is smaller than pin site one, that is, $B < B_p$, critical current density may depend on strength of the pin site force. In other words when $B > B_p$, pinned fluxoids cannot flow while Coffey has assumed that even fluxoids caught by pin sites can flow. Thus when $B < B_p$, in Coffey's theory B_p has been replaced by B simply in critical current density representation for $B > B_p$ but in our theory we should put

$$F_p = B \cdot C_p \tag{66}$$

where C_p is strength of a pin site force which will be constant for B.

3) Coffey has replaced h_{01} in Eqs. (9) and (10) by h_{02} to obtain j_3 that is contribution of fluxoids lattice, which will lead quantization condition Eq. (11) not to hold, and we have not done so. And the same time he has taken $N=1$, but we have treated N as parameter.

4) We have neglected both pinning loss, which may be caused by hysteresis of pinning and depinning fluxoids, and the loss which will be dissipated in normal and superconducting transition process in superconductor when flux flow is occurring.

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