Free Convection Heat Transfer from a Short Vertical Plate

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Abstract

Free convection heat transfer from a short vertical plate (with vertical length l and thickness d) was numerically analyzed by the finite difference method. The present results regarding average Nusselt number on a thin vertical plate ($d \simeq O$) are closely approximated by the following equation :

$$\overline{Nul} = 0.437 + 0.461 Grl^{1/4}, \; Pr = 0.72 \text{ and } 15 < Grl < 27000$$

This relation between $\overline{Nul}$ and Grl shows the same tendency as the experimental results and gives slightly lower Nusselt number.

In the range of l and d considered in the present study ($d/l \leqq 1$ etc.), using length $l + d$ in both Nusselt and Grashof numbers instead of l , average Nusselt numbers on the vertical plate with finite thickness can be approximated by the above correlation equation with an error within about 6%.

1. Introduction

The free convection heat transfer from a short vertical plate, when its Grashof number is less than 10^4, can not be analyzed by using the classical boundary layer approximation. It has not been investigated as fully as heat transfer around a thin horizontal circular cylinder. Then there are few publications on free convection heat transfer a short vertical plate at a very small Grashof number.

McAdams [1] proposed a recommended correlation curve giving the relation between an average Nusselt number and a Rayleigh number of the free convection heat transfer from a short vertical plate, which was based on the experimental results obtained by Saunders [2]. It was indicated by Fujii [3] and Ede [4] that the heat transfer coefficients obtained by experiment had usually a tendency to be higher because it was difficult to make an accurate estimate of the heat losses other than a free convection heat transfer. They also pointed out that the property values quoted by Saunders were not so accurate. Suriano and Yang [5] analyzed theoretically the free convection heat transfer from a short vertical plate at a very small Grashof number. But their results regarding the averaged Nusselt number showed a different tendency from the experimental results.

In this paper, finite difference method was used to analyze the free convection heat transfer from a short vertical plate at Grashof number between 10 and 10^5 for $Pr = 0.72$. The two thermal boundary conditions on the vertical plate which were considered in the present calculation were a uniform surface temperature condition and a uniform

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surface heat flux condition. On the former condition, the effects of the plate thickness and the horizontal ceiling over the vertical plate on the free convection heat transfer were numerically studied. The present numerical solutions demonstrated that the plate thickness had an influence on the local heat transfer coefficient near the leading edge, and that the effect of the horizontal ceiling were negligibly small at the larger dimensionless distance H_2 between the ceiling and the top of the plate than 20, when the dimensionless vertical length of the plate, L, *was* 20.

In the range of L and D considered in the present calculation ($5 \leqq L \leqq 30$, $D \leqq 10$, $D/L \leqq 1$), using length $l + d$ in both Nusselt and Grashof numbers instead of l, average Nusselt number on the vertical plate with finite thickness d could be approximated by the correlation equation for $D = 0$ with an error within 6 %.

2. Basic Equations and Numerical Solutions

A vertical plate and coordinate system are shown in Fig. 1. The basic equations

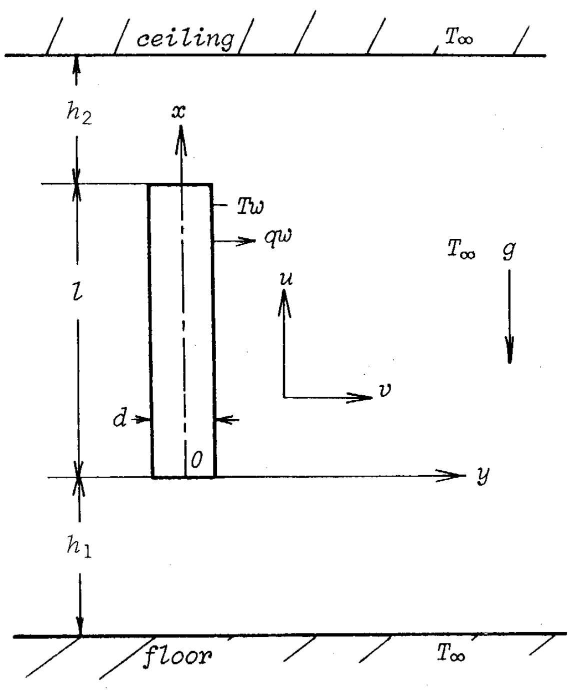

Fig. 1 A vertical plate and coordinate system.

which govern the two dimensional laminar free convection, are the equations of energy, vorticity and relation between vorticity and stream function. These equations become in a dimensionless form, using Boussinesq approximation,

$$\frac{\partial \theta}{\partial \tau} = -\frac{\partial U \theta}{\partial X} - \frac{\partial V \theta}{\partial Y} + \frac{1}{Pr} \nabla^2 \theta \tag{1}$$

$$\frac{\partial \xi}{\partial \tau} = -\frac{\partial U \xi}{\partial X} - \frac{\partial V \xi}{\partial Y} + \nabla^2 \xi - \frac{\partial \theta}{\partial Y} \tag{2}$$

$$\xi + \nabla^2 \psi = 0 \tag{3}$$

These dimensionless quantities are defined as follows :
1) Uniform surface temperature on the vertical plate

$$\tau = \frac{(g\beta\Delta T)^{2/3}}{\nu^{1/3}} t,\ X = \left(\frac{g\beta\Delta T}{\nu^2}\right)^{1/3} x,\ U = \frac{u}{(\nu g\beta\Delta T)^{1/3}},\ \theta = \frac{T - T_\infty}{\Delta T}$$

$$\Delta T = Tw - T_\infty,\ \xi = \frac{\nu^{1/3}}{(g\beta\Delta T)^{2/3}} \zeta,\ \psi = \frac{\phi}{\nu},\ P = \frac{p}{(\nu g\beta\Delta T)^{2/3}} \quad (4)$$

2) Uniform surface heat flux on the vertical plate

$$\tau = \left(\frac{g\beta qw}{\lambda}\right)^{1/2} t,\ X = \left(\frac{g\beta qw}{\nu^2\lambda}\right)^{1/4} x,\ U = \left(\frac{\lambda}{g\beta qw\,\nu^2}\right)^{1/4} u$$

$$\theta = \left(\frac{g\beta\lambda^3}{\nu^2 qw^3}\right)^{1/4} (T - T_\infty),\ \xi = \left(\frac{\lambda}{g\beta qw}\right)^{1/2} \zeta,\ \psi = \frac{\phi}{\nu} \quad (5)$$

, where t is time ; x is vertical distance from the leading edge of the vertical plate ; u is velocity in x direction ; T is temperature ; ζ is vorticity ; ϕ is stream function ; p is pressure ; g is acceleration due to gravity ; λ is thermal conductivity ; β is volume expansion coefficient ; ν is kinematic viscosity ; and qw is surface heat flux. The dimensionless forms of the length (l, d, y, h_1, h_2, n, s) are defined by the same way as x . The flow is considered to be symmetric about the vertical center plane of the vertical plate. The boundary conditions become

$$U = V = \psi = 0,\ \xi = -\frac{\partial^2\psi}{\partial N^2} \quad \text{and}$$

$\theta = 1$, for uniform surface temperature or

$$-\frac{\partial\theta}{\partial Y} = 1, \text{ for uniform surface heat flux} \quad (6)$$

on the impermeable vertical plate, and

$$\psi = V = \xi = \frac{\partial U}{\partial Y} = \frac{\partial\theta}{\partial Y} = 0 \quad (7)$$

on the symmetry lines. A horizontal floor was placed below the vertical plate. But its effects on the free convection heat transfer around a vertical plate are negligibly small because the dimensionless distance H_2 between the floor and the leading edge of the plate is larger than 40. [6] The boundary conditions are

$$U = V = \psi = \theta = 0,\ \xi = -\frac{\partial^2\psi}{\partial X^2} \quad (8)$$

on the horizontal floor. When a horizontal floor was not placed, the boundary conditions are

$$X=-\infty,\ 0<Y<\infty\ ;\ V=\frac{\partial^2\psi}{\partial X^2}=\theta=0,\ \xi=-\frac{\partial^2\psi}{\partial Y^2} \tag{8'}$$

The other outer boundary conditions become

$$X=\infty,\ 0<Y<\infty\ ;\ V=\frac{\partial^2\psi}{\partial X^2}=\frac{\partial\theta}{\partial X}=0,\ \xi=-\frac{\partial^2\psi}{\partial Y^2} \tag{9}$$

$$-H_1<X<\infty,\ Y=\infty\ ;\ U=\frac{\partial^2\psi}{\partial Y^2}=\theta=0,\ \xi=-\frac{\partial^2\psi}{\partial X^2} \tag{10}$$

When a horizontal ceiling is placed over the vertical plate, the boundary conditions are

$$U=V=\psi=\theta=0,\ \xi=-\frac{\partial^2\psi}{\partial X^2} \tag{11}$$

on the ceiling.

Basic equations (1), (2) and (3) with the above mentioned boundary conditions were numerically solved by the finite–difference ADI method using a variable mesh size. Further details of the numerical method are given in references [6] and [7]. After the converged solution of θ , ξ and ψ were obtained numerically, dimensionless pressure distributions were calculated by the following equation :

$$\nabla^2P-2\left[\frac{\partial V}{\partial X}\frac{\partial U}{\partial Y}-\frac{\partial U}{\partial X}\frac{\partial V}{\partial Y}\right]-\frac{\partial\theta}{\partial X}=0 \tag{12}$$

Then the boundary conditions are

$$\frac{\partial P}{\partial N}=\theta\frac{\partial N}{\partial X}-\frac{\partial\xi}{\partial S} \tag{13}$$

on the impermeable solid surface, and the other boundary conditions are the same form as that for θ .

The local Nusselt number Nux on the vertical surface are defined by the following equations.

$$Nux=\frac{\alpha x}{\lambda}=-X\frac{\partial\theta}{\partial Y}\Big|_{Y=D/2}\ \text{on the uniform temperature surface,}$$

$$Nux=\frac{\alpha x}{\lambda}=\frac{X}{\theta}\Big|_{Y=0}\ \text{on the uniform heat flux surface.} \tag{14}$$

In the case of uniform surface temperature, the effects of the finite thickness of the plate on the free convection heat transfer were studied. Then local Nusselt numbers Nud on the bottom and top horizontal surfaces of the vertical plate were defined by the following equations :

$$Nud = \frac{\alpha d}{\lambda} = D\frac{\partial \theta}{\partial X}\bigg|_{X=0} \text{ on the bottom surface,}$$

$$Nud = \frac{\alpha d}{\lambda} = -D\frac{\partial \theta}{\partial X}\bigg|_{X=L} \text{ on the top surface.} \tag{15}$$

Average Nusselt numbers $\overline{Nul}$ were defined by the following equations.

$$\overline{Nul} = \frac{\overline{\alpha} L}{\lambda} = \frac{L}{L+D}\left\{\int_0^L -\frac{\partial \theta}{\partial Y}\bigg|_{Y=D/2} dX + \int_0^D \left[\frac{\partial \theta}{\partial X}\bigg|_{X=0} - \frac{\partial \theta}{\partial X}\bigg|_{X=L}\right] dY\right\} \tag{16}$$

on the uniform temperature surface.

$$\overline{Nul} = \frac{\overline{\alpha} l}{\lambda} = \frac{L}{\int_0^L \theta\big|_{Y=0} dX} \tag{17}$$

on the uniform heat flux surface (a vertical surface only). The calculated results in the case of uniform surface heat flux can be compared with those in the case of uniform surface temperature by the following equation :

$$Grx^* = Nux\ Grx \tag{18}$$

where Grx^* is modified Grashof number, $(= X^4)$, Grx is Grashof number, $(= X^3)$.

3. Numerical Results and Discussions

Numerical solutions were computed on a FACOM M−200 in Kyushu University and for $Pr = 0.72$ and $H_1 > 40$. When horizontal ceiling is not placed over the vertical plate, the position of the outer boundary in the down stream direction is at $X = L + H_2$ and $H_2 > 20$. For a set of the given values of H_1, H_2, L, D and Pr, CPU time required was about 2 ~ 1 0 minutes.

3-1. Average Nusselt Number for a Vertical Thin Plate ($D = 0$)

The present numerical results of average Nusselt number for a thin vertical plate without ceiling are compared with the previous experimental and theoretical results in Fig. 2. The experimental results obtained by Saunders have been reevaluated by using the latest data of the physical properties at film temperature after the method of Fujii's study. [3] The difference between the reevaluated Saunders' results and McAdams' results (indicated by two−dotted chain line) can be considered to be caused by the differ-

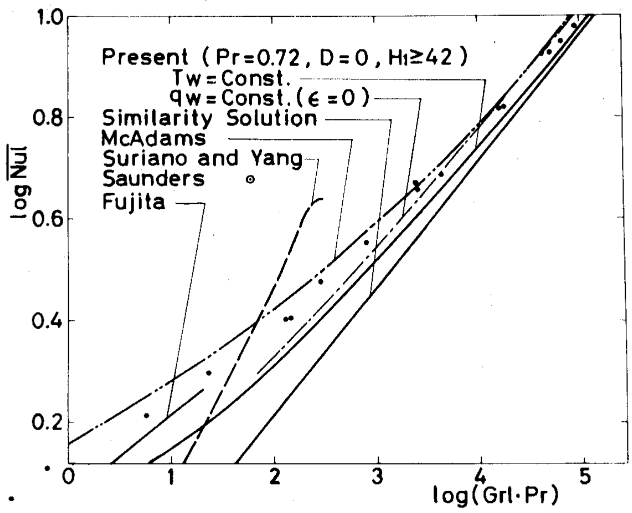

Fig. 2 Comparison of average Nusselt number on a thin vertical plate.

ence of the physical properties. The boundary condition on the heated surface in Saunders' experiment seems to be close upon the uniform heat flux condition. The present results for the uniform heat flux condition agree well with Saunders' results at larger Rayleigh number than $10^{3.5}$. The present solutions indicate that the difference of $\overline{Nul}$ caused by the difference of the thermal boundary conditions becomes smaller and the difference between the present theoretical result and Saunders' experimental result becomes larger, as Rayleigh number becomes smaller. The recent experimental result obtained by Fujita et al. [8] gives a value of $\overline{Nul}$ closer to the present study than that of Saunders'. But in all cases, the experiments give higher $\overline{Nul}$ than the present theoretical solutions at smaller Rayleigh number *(Grl Pr)*. The theoretical results obtained by Suriano and Yang have a different tendency from the experimental results. The present result of average Nusselt number on the uniform temperature surface is approximated closely by the following equation.

$$\overline{Nul} = 0.437 + 0.461\, Grl^{1/4}, \qquad Pr = 0.72 \tag{19}$$

Eq. (19) gives slightly smaller Nusselt number than Churchill and Chu correlation Eq. (20) . [9]

$$\overline{Nul} = 0.68 + 0.475\, Grl^{1/4}, \qquad Pr = 0.72 \tag{20}$$

3-2. The Effects of the Plate Thickness on the Free Convection Heat Transfer

The local Nusselt number distributions on the vertical plate with various dimensionless thickness D and the same dimensionless vertical length L are compared in Figs. 3 and 4, respectively. The distributions shown in the left and right hand sides of each figure correspond to the local Nusselt numbers on the bottom and the top horizontal sur-

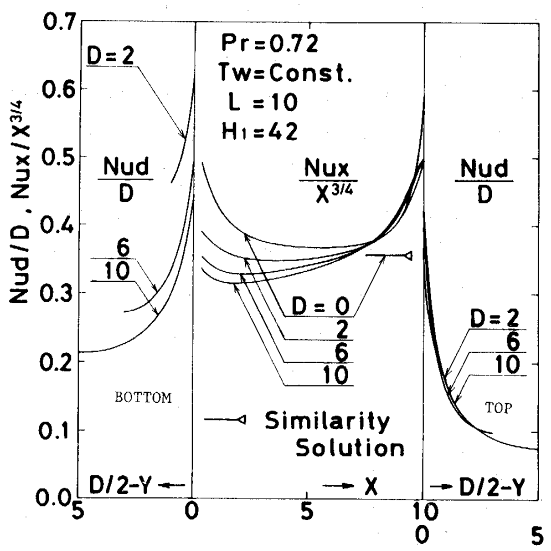

Fig. 3 Local Nusselt number distributions on a short vertical plate with finite thickness, $L = 10$.

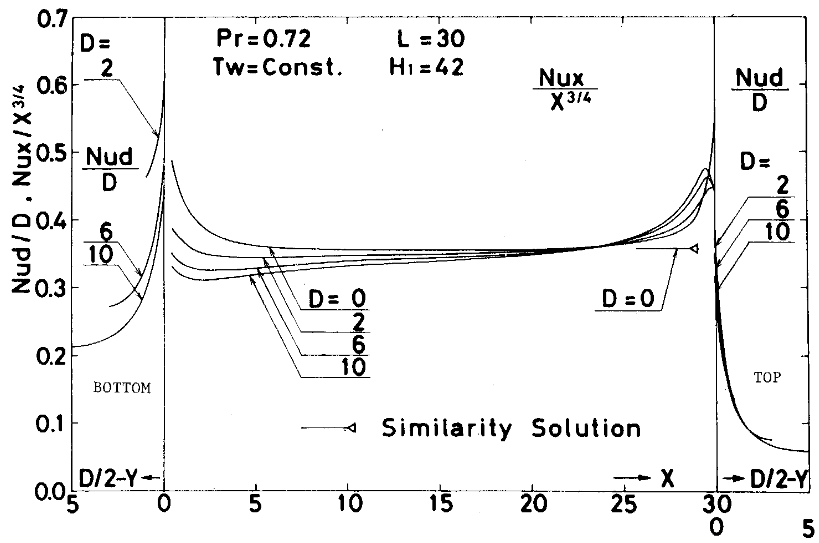

Fig. 4 Local Nusselt number distributions on a short vertical plate with finite thickness, $L = 30$.

faces of the thick vertical plate, respectively. The local Nusselt number on the vertical surface (shown in the central part of each figure) are divided by $X^{3/4}$ to be compared with the similarity solution. Each figure indicates that the larger dimensionless thickness of the vertical plate with the constant dimensionless vertical length gives the lower local Nusselt number on its bottom surface and on the lower part of its vertical surface. The local Nusselt number on its top surface and on the upper part of its vertical surface are hardly influenced by the change of D. Furthermore, comparison between these two figures indicates that the local Nusselt numbers on the bottom surface and on the lower part of the vertical surface are almost unaltered by the change of L, if the plate dimensionless thickness D is constant. When D is larger than 2, the local Nusselt numbers on the vertical surface become partly lower than the similarity solution. [3]

In Fig. 5 the present numerical results of the average Nusselt number on the vertical plate with various L (= 30, 20, 10, 5 and 2.5) and D (= 10, 6, 2 and 0) are plotted against L and Grl. The previous theoretical results for a horizontal cylinder [10], [11] and the similarity solution of a vertical surface are also shown for comparison. The dimensionless thickness D and the aspect ratio D/L (is lower than 1) of the vertical plate are

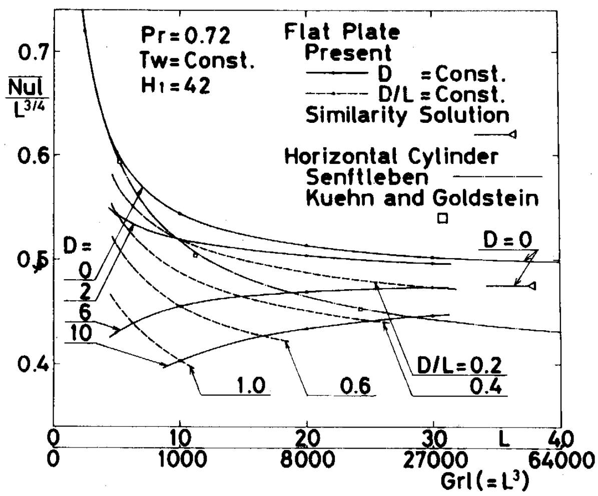

Fig. 5 Average Nusselt number on a short vertical plate with finite thickness.

Table 1 Calculated average Nusselt number on a short vertical plate

			a	b	
L	D	$\overline{Nul}$	$\overline{Nuld}$	$0.437+0.461\,(L+D)^{3/4}$	$(a-b)/b$ %
30	0	6.37	6.37	6.35	0.03
30	2	6.36	6.78	6.64	2.2
30	6	6.06	7.27	7.21	0.8
30	10	5.72	7.63	7.77	−1.8
20	0	4.79	4.79	4.80	−0.2
20	2	4.76	5.23	5.12	2.2
20	6	4.43	5.76	5.75	0.3
20	10	4.10	6.15	6.35	−3.1
10	0	3.00	3.00	3.03	−1.0
10	2	2.92	3.50	3.41	2.6
10	6	2.56	4.09	4.13	−0.8
10	10	2.27	4.53	4.80	−5.6
5	0	1.96	1.96	1.98	−1.0
5	2	1.82	2.54	2.42	5.0
5	6	1.44	3.16	3.22	−1.8
2.5	0	1.39	1.39	1.35	3.0

maintained constant along the solid and broken lines shown in this figure, respectively. The larger dimensionless thickness D of the plate with the same L gives the smaller average Nusselt number. The difference of average Nusselt number between the present result on thin vertical plate ($D = 0$) and that on a horizontal cylinder becomes negligibly small when L is lower than 5.

In the range of L and D in the present calculation, using length $l + d$ in the both Nusselt and Grashof numbers instead of l, present average Nusselt numbers on the vertical plate with finite thickness d can be approximated by Eq. (19) with an error within 6 %, as shown in Table 1. On the other hand, using length l in the both Nusselt and Grashof numbers, present average Nusselt numbers on the vertical plate with finite thickness become 27 % (at the maximum) lower than Eq. (19).

3-3. The Effects of the Horizontal Ceiling on the Free Convection Heat Transfer

In Fig. 6 the local Nusselt number distributions on the thin vertical plate ($L = 20$

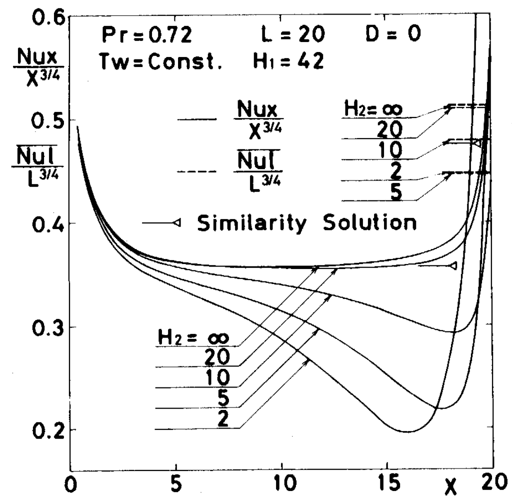

Fig. 6 Nusselt numbers on a thin vertical plate under a ceiling.

and $D = 0$) are shown, when the horizontal ceiling over the plate is placed at the various heights from the trailing edge of the plate. At the right hand side in Fig.6 average Nusselt numbers are plotted also by the broken lines. All these Nusselt number are divided by $X^{3/4}$ or $L^{3/4}$ to be compared with the similarity solution of the free convection vertical boundary layer. The smaller value of H_2, which is the dimensionless height of the ceiling from the top of the plate, gives the lower Nusselt number. But local Nusselt number distribution very near the leading edge is hardly reduced by a decreasing of H_2. The average and local Nusselt numbers for $H_2 = 20$ are close to these on the vertical plate without a ceiling. For $H_2 = 2$, local Nusselt number very near the trailing edge becomes very large by the heat conduction because the trailing edge at temperature Tw approach closely to the ceiling at temperature T_∞.

3-4. Velocity, Temperature and Pressure Fields around a Vertical Plate

The distributions of streamline, isothermal line and isobaric line around the thin vertical plate ($L = 20$ and $D = 0$) are shown in Figs. 7, 8 and 9. The dimensionless

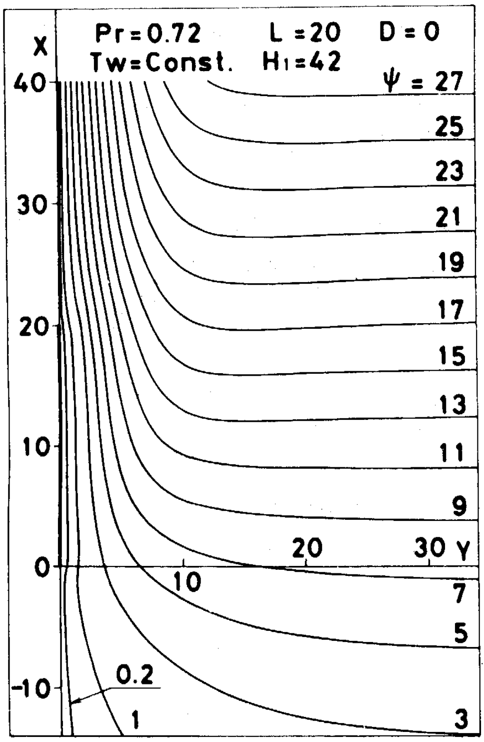

Fig. 7 Distribution of streamline around a thin vertical plate.

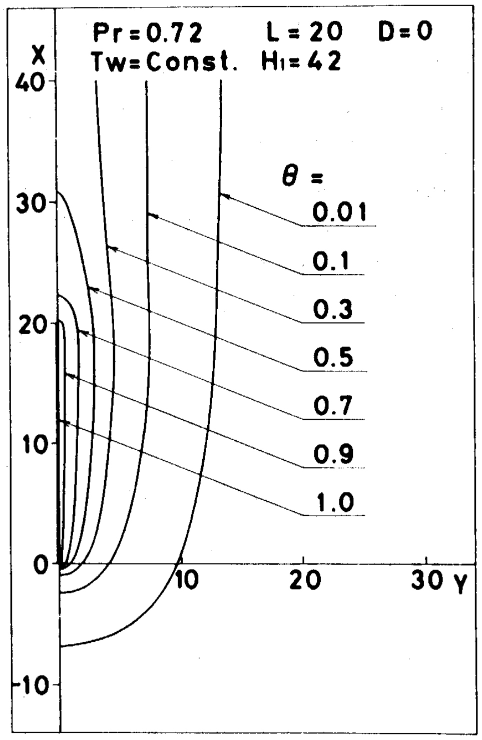

Fig. 8 Distribution of isothermal line around a thin vertical plate.

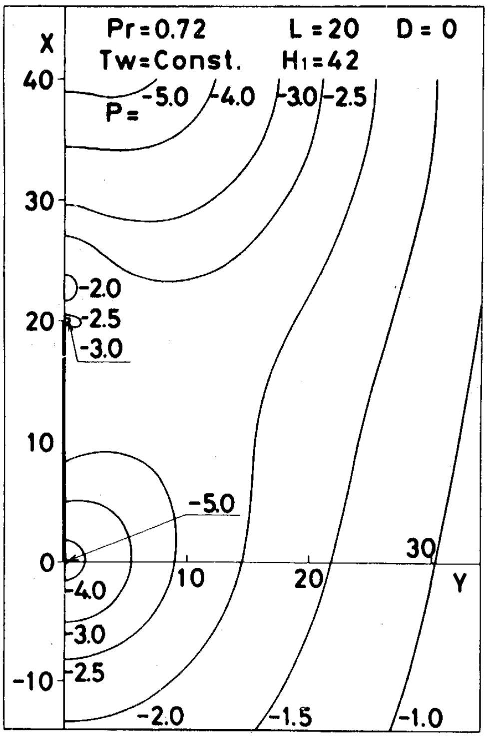

Fig. 9 Distribution of isobaric line around a thin vertical plate.

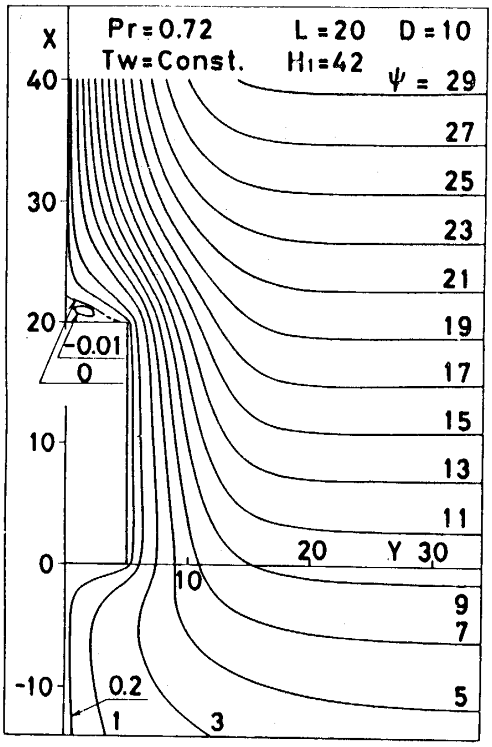

Fig. 10 Distribution of streamline around a vertical plate with finite thickness.

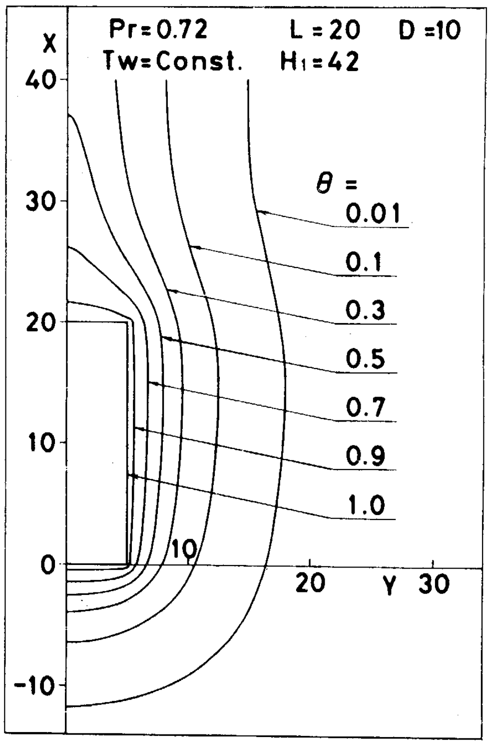

Fig. 11 Distribution of isothermal line around a vertical plate with finite thickness.

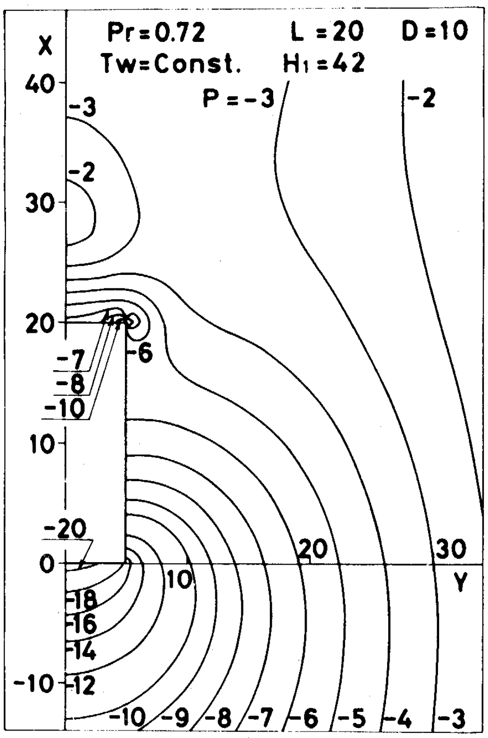

Fig. 12 Distribution of isobaric line around a vertical plate with finite thickness.

pressure is lowest near the leading edge. The streamline pattern under the leading edge indicates that there is a flow drawn into the sink at the leading edge.

When $L = 20$ and $D = 10$, these distributions are shown in Figs. 10, 11 and 12. There is a separation bubble at the top of the plate. This separation bubble disappears in the case of the shorter vertical plate with the same thickness ($L = 10$ and $D = 10$).

4. Conclusions

1. The present results regarding average Nusselt number on a thin vertical plate are closely approximated by the following equation :

$$\overline{Nul} = 0.437 + 0.461\ Grl^{1/4},\ Pr = 0.72,\ 15 < Grl < 27000$$

2. This relation shows the same tendency as the experimental results and gives slightly lower Nusselt number.
3. The larger dimensionless thickness D of the vertical plate with the constant dimensionless height L gives the lower local Nusselt number especially on the bottom surface and on the lower part of the vertical surface, and the larger thickness gives the lower average Nusselt number.
4. When the dimensionless thickness D is constant, the local Nusselt numbers on the bottom surface and the lower part of the vertical surface are almost constant, even if the dimensionless height L is altered.
5. In the range of L and D in the present calculation, ($5 \leqq L \leqq 30$, $D \leqq 10$, $D/L \leqq 1$),using length $l + d$ in both Nusselt and Grashof numbers instead of l, average Nusselt numbers on the vertical plate with thickness d can be approximated by the correlation equation for the thin vertical plate ($d = 0$) with an error within 6 %.
6. The effects of the ceiling over the plate on the free convection heat transfer are negligibly small at the larger dimensionless height H_2 of the ceiling from the plate trailing edge than 20, when L is 20 and D is 0.

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