On the Velocity Distribution of Open Channel Flow with Large Relative Roughness

By Takashi SAITOU* and Takeshi OKADA**

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Abstract

The purpose of this paper is to make clear the displacement height, and the thickness of roughness region in which the logarithmic equation cannot be applied, as aclue to elucdate the roughness region, and continue investigating the applicability of logarithmic resistance equation in such case.

In this paper, we report the results of the experiment on the effect of relative roughness on the velocity distribution in open channel and discuss on the displacement height and the thickness of roughness region with change of constant value of logarithmic resistance equation and turbulent viscosity. A conclusion cannot be drawn due to an insufficient number of experimental samples, it seems to be quite all right to consider that there are effect of relative roughness and river bed slope on the displacement height and the thickness of roughness region.

Introduction

The greater parts of investigation on the resistance to flow in river have concentrated on flow with small relative roughness. The resistance to flow in open channel with large relative roughness have received a little attension. However, schemes far flood routing and erosion contral works have come to involve the prediction of velocity of flow in river, such as those in mountainous area, where conditions of large scale roughness exist. Therefore, the resistance equation covering such river is strongly required.

Bathurst[1] reported that the roughness cannot be considered small scale at relative roughness $k/h>0.3$, in which k = the height of the roughness and h = the depth of flow. The studies of Hartung and Scheuerlein[2] Scheuerlein[3] and Ashida and Bayazit[4] make it clear that the resistance equation for small scale roughness do not apply at large relative roughness. Bayazit[5] founded that once the relative roughness exceeds $k/h>0.3$, the resistance to the flow is higher than that predicted by the logarithmic resistance equation. And, Herbich and Shulits[6] and Adachi[7] suggested that the resistance to flow is related to the pattern and spacing of the roughness elements. Yamaoka[8] has studied the charactristic of roughness of natural river channels by comparing with the resistance of channels the rectangular roughness elements.

As mentioned above, in the case of relative roughness is small, the logarithmic resistance equation is applicable to the most of the flow. But, as the value of relative roughness increase, the resistance to the flow is higher than that predicted by the logarithmic resistance equation, and the difference of both becomes greater as the value of relative

* Department of Civil Engineering
** Graduate student, Civil Engineering

roughness increase further. It is satisfactory to consider the difference of both as the complicated structure of flow at vicinity of roughness, which are replaced by the displacement height in leading the logarithmic resistance equation, cannot be neglected, since the roughness region become a large part of flow as relative roughness increase. In use of logarithmic resistance equation, it is considered necessary to make clear the displacement height and the thickness of the roughness region. And, it seems likely that clarification of these is one of advantageous method to investigate the generalized expression of resistance to flow in open channel. So that, in this paper, we examined how the value of Ar and κ, which are constants of the logarithmic velocity distribution law, changes in direction of depth by using local grade of velocity, when the displacement height changes systematically.

Experimental Apparatus and Measurements

The experiments are conducted in 40cm wide by 25cm deep rectangular flume which meaured 8.6m in length. The flume and water circulation system are shown schematically in Fig. 1. The roughness elements which are fixed densely on channel bed, are lightweight aggregate with similar form to natural sand grade. The mean diameter of roughness elements is d=1.35cm. The compasition of roughness elements are fairly uniform.

Experiments are conducted at discharges varying from 1 l/sec to 6.7 l/sec and at the slope of 0.005 and 0.0103. For measurement of velocity, we used a pitot tube which were made of an alminous pipe with the outside diameter of 12mm and flattened to 14.5 mm wide 1mm thickness at its mouth, in order to measure transverse mean velocity at vicinity of roughness elements.

For the measurement of water depth, we used thee pitot static pressure tubes, which were made of strainless pipe with the outside diameter of 2mm and the inner diameter

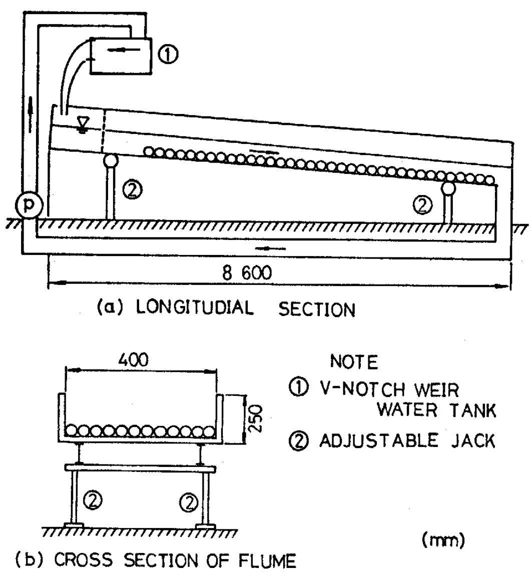


Fig. 1 Flume and circulation system.

of 1mm. In order to measure pressure difference, which were necessary to measurement of velocity and water level with high accuracy, we employed balance type difference meter[9].

Experimental Results

The well-known equation of the logarithmic velocity distribution for roughness region is as follow.

$$\frac{U}{U_*}=Ar+\frac{1}{\kappa}\ ln\ \left(\frac{Y}{ks}\right) \qquad \cdots(1)$$

Where U is the velocity at a vertical distance Y from wall, U_* is the shear velocity, ks is the equivalent roughness, the value of κ is 0.4 as Kármán's universal constant, and the value of Ar is 8.5.

The shear velocity U_* is given as follow.

$$U_*=\sqrt{g\ h\ I} \qquad \cdots(2)$$

where g is the acceleration of gravity, h is the water depth and I is the slope of flume.

Idealized two dimensional model and definition of the displacement height, namely the height of virtual wall, are shown in schematic form in Fig. 2. If we measured water

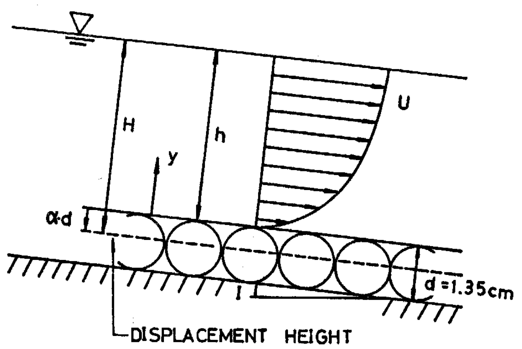


Fig. 2 Sketch of indealized two dimensional model and definition of the displacement hight.

depth and reference point from the height of virtual wall which locate αd below roughness crest, Eq. (1), (2) are rewritten as follow.

$$\frac{U}{U_*}=\ Ar\ +\frac{1}{\kappa}\ ln\left(\frac{Y+\alpha d}{ks}\right)$$
$$U_*=\sqrt{g\ (h+\alpha d)\ I} \qquad \cdots(3)$$

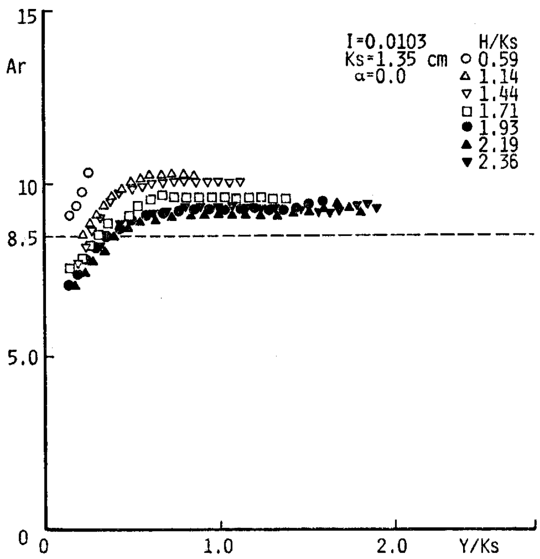
We examine how the value of Ar and κ change in direction of depth by above equation with measured velocity distribution, when the displacement height changes systematically.

The Value of Ar and κ

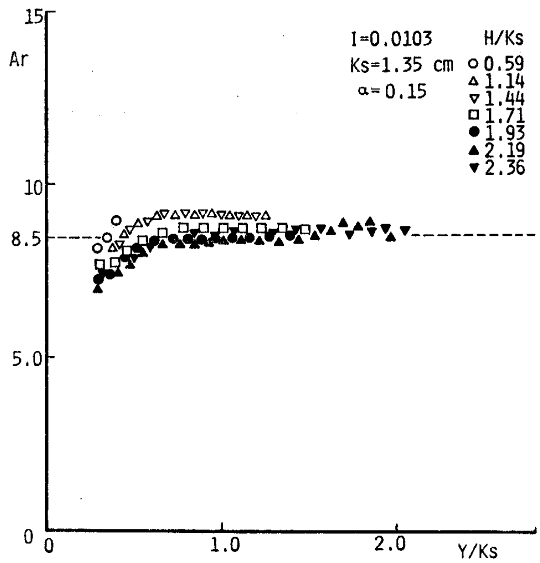
The value of Ar and κ were calculated as follow.

Using the least square method to nine measured values which were $U/\sqrt{g\ (h+\alpha d)\ I}$ to $(y+\alpha d)/d$, we determined local velocity distribution as curve of second degree. Then, we determined a tangent line which touch the curve of second degree at center point of nine measured points. Subsequently, we calculated the value of Ar from an intersection of this tangent line and horizontal axis and the value of κ from an inline of this tangent line.

In above calculation, we changed the value of α, which defined the displacement height, as follow.



(a)



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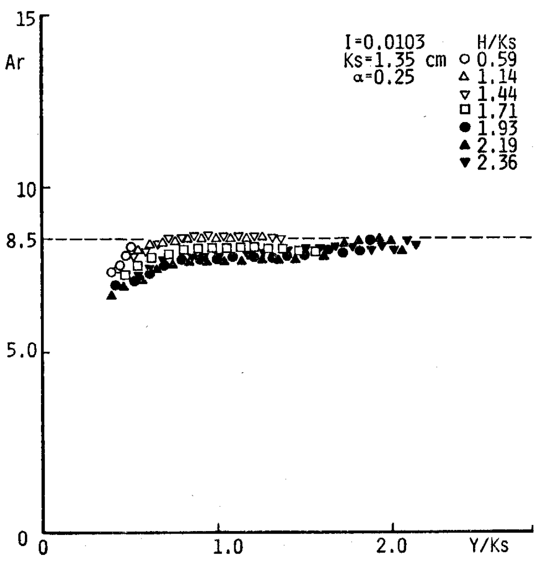

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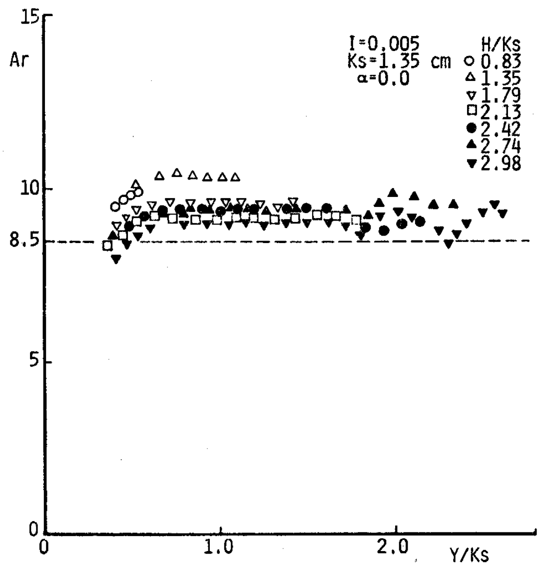

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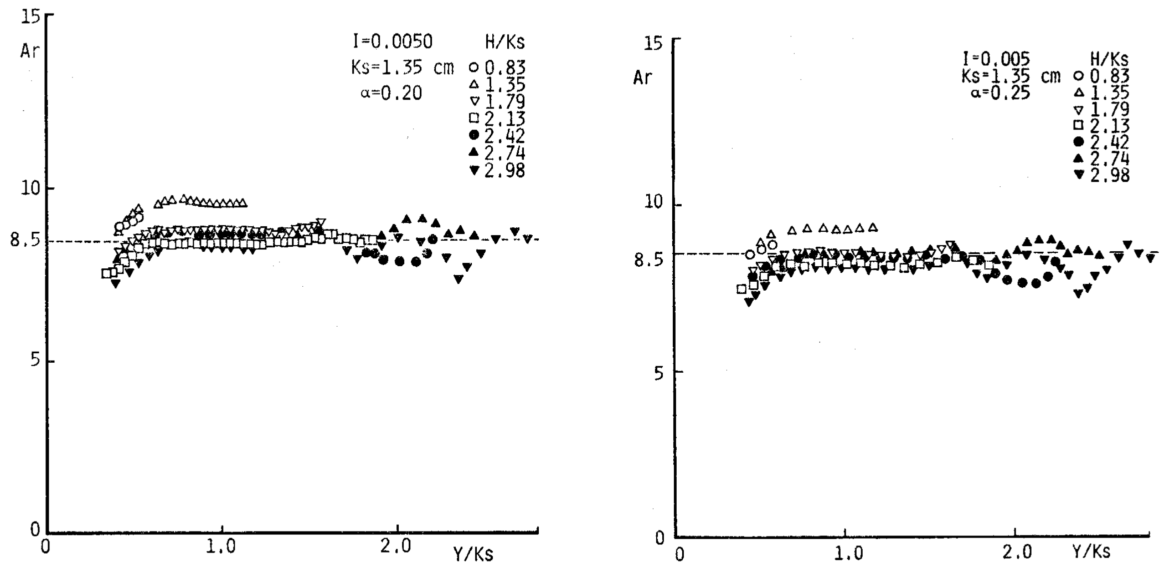


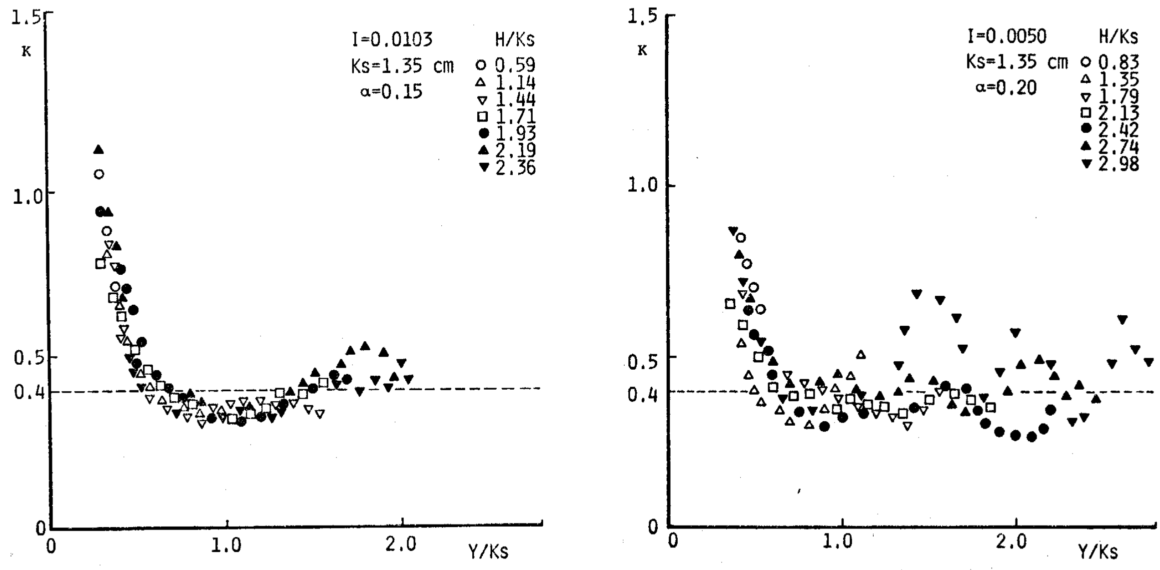
(e) (f)

Fig. 3 Relation between Y/ks and Ar

$\alpha=0.0,\ 0.1,\ 0.15,\ 0.20,\ 0.25,\ 0.50,\ 0.75,\ 1.0$

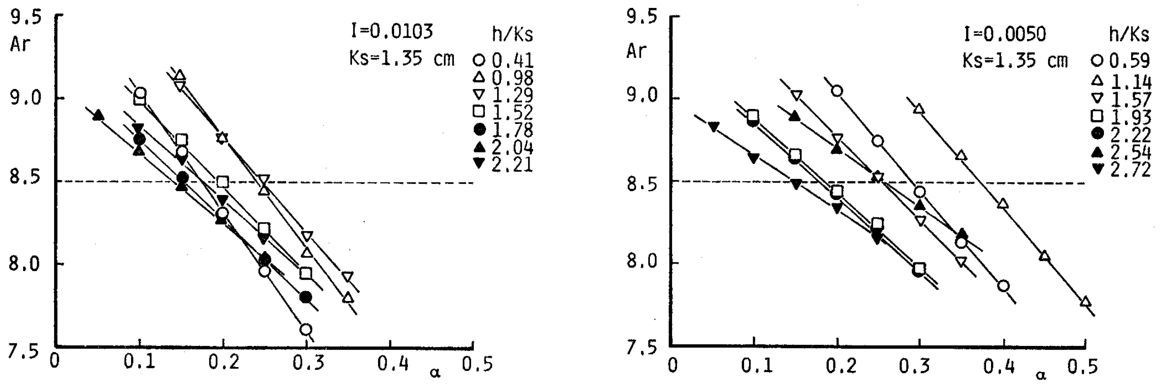
The typical example of the calculated values of Ar and K, which were obtained in the method decreased above, are shown in Fig. 3 and Fig. 4.

As it is clear from Fig. 3, the value of Ar increases as Y/ks increases in region Y/ks<0.6, but the value is almost constant in region Y/ks>0.6. It can be considered from this fact as follow. Using suitable value of α, the logarithmic velocity distribution can be applicable in the region Y/ks>0.6, but in the region Y/ks<0.6, it is necessary to give the value of α in function of Y/ks at least.



(a) (b)

Fig. 4 Relation between Y/ks and κ



(a) (b)

Fig. 5 Relation between the average of Ar in the region Y/ks>1.0 and the value of α.

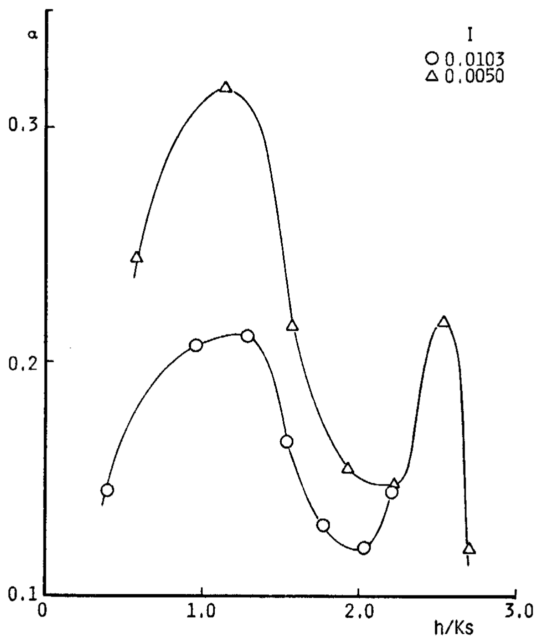


Fig. 6 Relation between the value of α and h/ks

Taking notice of the value of Ar in the region Y/ks>0.6, as the value of α and relative roughness ks/H decrease, the value of Ar increases. And, when applying the logarithmic velocity distribution to flow in the region Y/ks>0.6, the suitable value of α varies with slope of open channel. Accordingly, when applying the logarithmic velocity distribution to open channel flow, it is necessary to clear the relation between the value of α and ks/H, I and etc..

The value of κ which was obtained in the way described above are shown in Fig. 4. As is seen in the value of Ar, it seems to be quite all right to consider that the value of κ is almost constant in the region Y/ks>0.6.

The relation between the average of Ar in the region Y/ks>1.0 and the value of α is shown in Fig. 5 (a), (b). As is evident from Fig. 4, the value of Ar obtained in one experiment decrease linearly as the value of α increases. In Fig. 5, determine the value of α correspond to Ar=8.5, the relation of these value of α and h/ks is shown in Fig. 6.

A conclusion cannot be drawn due to an insufficient number of experimental samples, it seems to be quite all right to consider that there are effects of slope and h/ks an the value of α which gives the height of virtual wall. From the results on the value of α mentioned above, it is expected that the value of α is in form as follow.

$$\alpha = f \left(\frac{h}{ks}, \ I \right) \qquad \cdots(4)$$

The Coefficient of Eddy Viscosity

Strickly speaking, a shear stress distribution in the vicinity of large scale roughness does not indicate linear distribution in the effect of the action of inertial forces, but after the experimental reports by Kurika, it is possible to be considered that the shear stress distribution curve is linear ın this case practically.

So an equation of shear stress is assumed as follow.

$$\frac{\tau}{\rho} = \frac{\tau_0}{\rho} (1-n) = U_*^2 \ (1-\eta), \quad \eta = \frac{Y}{h} \qquad \text{—(5)}$$

where τ_0 denotes a shear stress of the wall surface.

In the steady flow of an incompressible fluid, if we neglect the term of an inertical force which is considered to be very small, Eulerian equation can be written in the following form.

$$\frac{\tau}{\rho} = \nu \frac{dU}{dY} - \overline{u'v'} = (\nu + \varepsilon) \frac{dU}{dY} \qquad \text{—(6)}$$

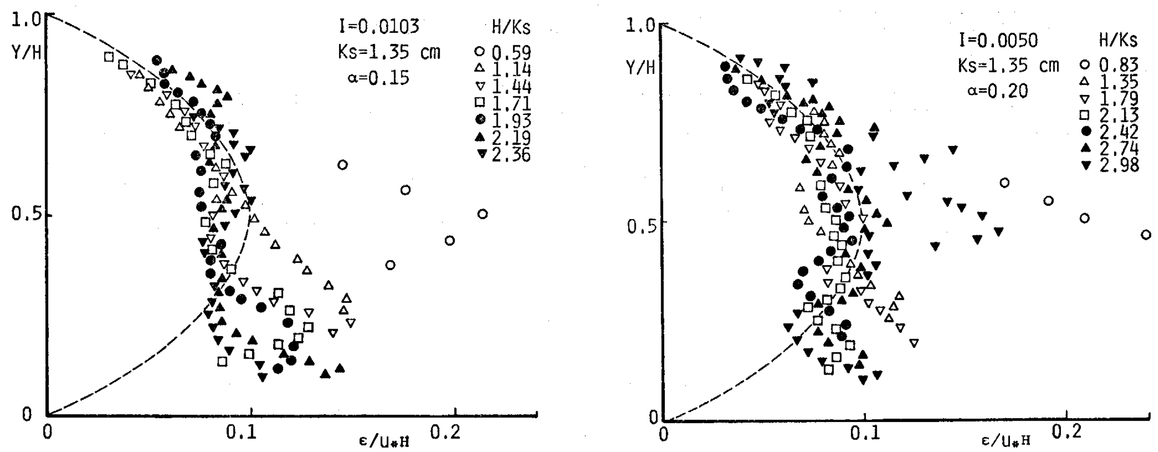
where ε is the coefficient of eddy viscosity. In the neighborhood of the wall surface, ν is far smaller than ε, so that ν can be neglected. Finally, by using Eq. (5), (6) the coefficient of eddy viscosity is written in the following form.

$$\frac{\varepsilon}{U_* h} = \frac{U_*(1-\eta)}{h} \left(\frac{dU}{dY} \right) \qquad \text{—(7)}$$

In the turbulent mixing region of flow, where logarithmic velocity distribution can be apllied as mentioned above, then dU/dY is discribed as $U_*/\kappa Y$, so that Eq. (7) is rewritten in the following form.

$$\frac{\varepsilon}{U_* h} = \kappa \eta \ (1-\eta) \qquad \text{—(8)}$$

Using the least square method to nine measured values in much the same way as the calculation of Ar, we determined a tangent line which touch the curve of second degree at center point of nine measured points. Then, we calculated coefficient of eddy viscosity by Eq. (7) in substituting the value of dU/dY decided above.



(a) (b)

Fig. 7 Comparison of the value of eddy viscosity, calculated by Eq. (7) and measured valocity distribution with Eq. (8)

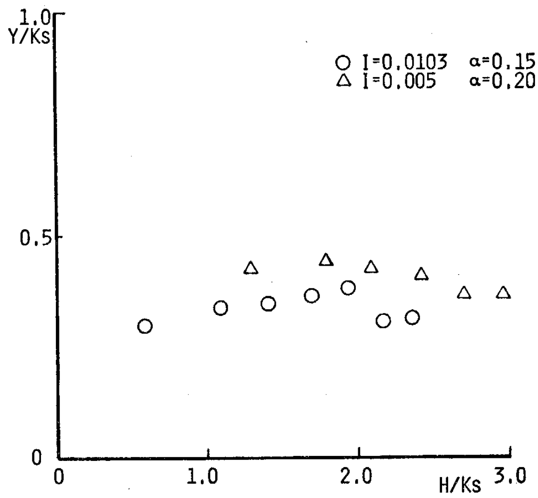


Fig. 8 The largest value of ε/u_*H

The comparison of the value of eddy viscosity calculated by Eq. (7) and measured velocity distribution with Eq. (8) are shown Fig. 7. The value of ε/U_*H shows a very interesting tendency as follow, namely in the region $Y/H>ks/H$, the value of ε/U_*H calculated by Eq. (7) and measured velocity distribution almost concern with Eq. (8), but in the region $Y/H<ks/H$, the former is greater than the latter.

The position where the value of ε/U_*H is the largest in the roughness region are shown in Fig. 8. It seems quite all right to consider that there is effect of river bed slope on the position of maximum eddy viscosity in the roughness region, and there is little effect of relative roughness.

Conclusion

The summary of the experimental conclusion concerning the velocity distribution in open channels with large scale roughness is as follow.

1) The law of velocity distribution is not applicable to flow in the region (Y/ks< 0.6), namely roughness region.

2) In order to apply the law of velocity distribution to the turbulent mixing region (Y/ks>0.6), it is necessary to introduce the displacement height. It is considered that the displacement height is a function of the relative roughness and the slope of river bed.

3) In the roughness region (Y/ks<0.6), the hydraulic charactristic is an later problem, however, the thickness of it is considered nearly stable.

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