Thermal Radiation with Conduction and Convection

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Abstract

The author accomplished the mathematical analyses on the case of which hot gas flows through inside of a pipe with multiple concentric layers, and thermal radiation, heat conduction and forced convection coexist at the inside and outside of the pipe at the same time.

It is possible to linearize the nonlinear boundary conditions by means of Legendre's polynomials on the whole interval.

The simultaneous Integrodifferential equations and the simultaneous Volterra's integral equations are reduced to the simultaneous algebraic equations.

The finite Hankel Transforms are practised over the paper with emphasis.

1. Introduction

High temperature heat transfer devices are often associated with large convection and radiant heat. The thermal radiation becomes of greater importance as the temperature level increases[1].

The developments of researches on the case of which thermal radiation, heat conduction and heat transfer coexist at the same time are very necessary, but the researches on these cases are extremely difficult theoretically or experimentally.

The thermal radiation characteristics of enclosures are a matter of considerable practical interest[2,3,4].

The author accomplished the theoretical and mathematical analyses on the case of which hot gas flows through inside of a pipe with multiple concentric layers, and thermal radiation, heat conduction and forced convection coexist at the inside and outside of the pipe at the same time.

2. Mathematical Analyses

Hot gas flows through the inside of multiple layers of pipes, heat is transferred by heat conduction in the walls of pipes, and thermal radiation and heat transfer occur at the inside and outside of pipes.

To write dimensionless forms, put

$$\theta_n=\frac{T_n}{T_o},\quad x_n=\frac{r+r_n}{r_{n+1}+r_n},\quad D_1=2r_1,$$

$$z=\frac{Z}{D_1},\quad (n=1,\ 2,\ 3)$$

(1) Equation of heat conduction for inside domain 1.

$$\frac{\partial^2\theta_1}{\partial x_1^2}+\frac{1}{x_1-r_1/(r_2+r_1)}\cdot\frac{\partial\theta_1}{\partial x_1}+\left(\frac{r_2+r_1}{D_1}\right)^2\cdot\frac{\partial^2\theta_1}{\partial z^2}=0 \text{ or } \nabla_1^2\theta_1=0 \tag{1}$$

$$-(\partial\theta_1/\partial x_1)_{x_1=\varepsilon_1}=(-a\theta_1^4-b\theta_1+d)_{x_1=\varepsilon_1},\quad \varepsilon_1=2r_1/(r_2+r_1) \tag{2}$$

(included radiation and heat transfer)

$$(\theta_1)_{x_1=1}=f_o(z),\quad (f_o(z)=F(Z)/T_o) \tag{3}$$

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$$\left(\frac{\lambda_1}{\lambda_2}\right)\left(\frac{\partial\theta_1}{\partial x_1}\right)_{x_1=1} = \left(\frac{r_2+r_1}{r_3+r_2}\right)\left(\frac{\partial\theta_2}{\partial x_2}\right)_{x_2=\varepsilon_2},\quad \varepsilon_2=2r_2/(r_3+r_2) \tag{4}$$

Put

$$\log_e \Phi_1=\int\frac{d\theta_1}{(-a\theta_1^4-b\theta_1+d)} \tag{5}$$

and

$$\Phi_1=a_n\cdot\theta_1+b_n \tag{6}$$

$$\Phi_1=u_1+\left(\frac{x_1-\varepsilon_1}{1-\varepsilon_1}\right)^2\cdot f(z) \tag{7}$$

$$f(z)=(\Phi_1)_{x_1=1}=a_n\cdot f_0(z)+b_n$$

Constant d contains gas temperature T_g, heat transfer coefficient and radiation constant[2].

Constants a_n and b_n are approximated in (27) in all parts over the interval $(\alpha_1,\ \beta_1)$ by Legendre's polynomials P_n.

From (1), (2) and (3), we obtain the next equations.

$$\nabla_1^2 u_1+Q_1(x_1,\ z)=0 \tag{8}$$

$$-\left(\frac{\partial u_1}{\partial x_1}\right)_{x_1=\varepsilon_1}=(u_1)_{x_1=\varepsilon_1} \tag{9}$$

$$(u_1)_{x_1=1} \tag{10}$$

$$Q_1(x_1,\ z)=\frac{2f(z)}{(1-\varepsilon_1)^2}\left\{1+\frac{x_1-\varepsilon_1}{x_1-\frac{r_1}{r_2+r_1}}\right\}+\left(\frac{x_1-\varepsilon_1}{1-\varepsilon_1}\right)^2\left(\frac{r_2+r_1}{D_1}\right)^2\frac{\partial^2 f(z)}{\partial z^2} \tag{11}$$

To solve (8), (9) and (10), the finite Hankel Transform[5] are applied.

The Eigen-values $\xi_{i,1}$ are to be determined by the given boundary conditions (9).

$$\cot\xi_{i,1}=\xi_{i,1}$$

From (8), (9) and (10), applying the finite Hankel Transform[5], we obtain the solution of u_1.

(2) Equation of heat conduction for middle domain 2.

$$\nabla_2^2 u_2+Q_2(x_2,\ z)=0 \tag{12}$$

$$(u_2)_{x_2=\varepsilon_2}=0 \tag{13}$$

$$(u_2)_{x_2=1}=0 \tag{14}$$

$$\theta_2=u_2+\left\{\Psi_0(z)-f_0(z)\right\}\left(\frac{x_2-\varepsilon_2}{1-\varepsilon_2}\right)+f_0(z),\quad \Psi_0(z)=\frac{\Psi(Z)}{T_0},$$

$$Q_2(x_2,\ z)=\frac{1}{\left(x_2-\frac{r_2}{r_3+r_2}\right)(1-\varepsilon_2)}\left\{\frac{\Psi(z)}{a'_n}-\frac{f(z)}{a_n}-\frac{b'_n}{a'_n}+\frac{b_n}{a_n}\right\}$$
$$+\left(\frac{r_3+r_2}{D_1}\right)^2\cdot\left[\left\{\frac{1}{a'_n}\cdot\frac{\partial^2\Psi(z)}{\partial z^2}-\frac{1}{a_n}\cdot\frac{\partial^2 f(z)}{\partial z^2}\right\}\frac{(x_2-\varepsilon_2)}{(1-\varepsilon_2)}+\frac{1}{a_n}\cdot\frac{\partial^2 f(z)}{\partial z^2}\right]$$

From (12), (13) and (14), applying the finite Hankel Transform[5], we can obtain the solution of u_2 similarly.

The Eigen-values $\xi_{i,2}$ are obtained from (14).

$$\xi_{i,2}=\frac{(2i-1)}{2}\cdot\pi \qquad (i=1,\ 2,\ 3,\ \cdots\cdots)$$

(3) Equation of heat conduction for outside domain 3.

$$\nabla_3^2\theta_3=0 \tag{15}$$

$$-\left(\frac{\partial\theta_3}{\partial x_3}\right)_{x_3=1}=(a'\theta_3^4+b'\theta_3-d')_{x_3=1} \tag{16}$$

(included radiation and heat transfer)

$$(\theta_3)_{x_3=\varepsilon_3}=\Psi_o(z),\quad \varepsilon_3=2r_3/(r_4+r_3) \tag{17}$$

$$\left(\frac{\lambda_2}{\lambda_3}\right)\left(\frac{\partial\theta_2}{\partial x_2}\right)_{x_2=1}=\left(\frac{r_3+r_2}{r_4+r_3}\right)\left(\frac{\partial\theta_3}{\partial x_3}\right)_{x_3=\varepsilon_3} \tag{18}$$

Put

$$\log_e \Phi_3=\int\frac{d\theta_3}{(a'\theta_3^4+b'\theta_3-d')} \tag{19}$$

$$\Phi_3=a'_n\theta_3+b'_n \tag{20}$$

$$\Phi_3=u_3+\frac{(1-x_3)^2}{(1-\varepsilon_3)^2}\cdot\Psi(z) \tag{21}$$

$$\Psi(z)=a'_n\Psi_o(z)+b'_n$$

Constants a'_n and b'_n are approximated in (27) in all parts over the interval $(\alpha_3,\ \beta_3)$ by Legendre's polynomials P_n.

From (15), (16) and (17), we obtain the next equations.

$$\nabla_3^2 u_3+Q_3(x_3,\ z)=0 \tag{22}$$

$$-\left(\frac{\partial u_3}{\partial x_3}\right)_{x_3=1}=(u_3)_{x_3=1} \tag{23}$$

$$(u_3)_{x_3=\varepsilon_3}=0 \tag{24}$$

$$Q_3(x_3,\ z)=\frac{2\Psi(z)}{(1-\varepsilon_3)^2}\left\{1-\frac{(1-x_3)}{\left(x_3-\dfrac{r_3}{r_4+r_3}\right)}\right\}+\left(\frac{r_4+r_3}{D_1}\right)^2\cdot\left(\frac{1-x_3}{1-\varepsilon_3}\right)^2\cdot\frac{\partial^2\Psi(z)}{\partial z^2}$$

From (22), (23) and (24), applying the finite Hankel Transform[5], we can obtain the solution of u_3 similarly.

The Eigen-values $\xi_{i,3}$ are obtained from (23). $\xi_{i,3}=\xi_{i,1}$.

From (5) and (19), we can put as follows generally.

$$\log_e \Phi_n=\int\frac{d\theta_n}{(K-B\cdot\theta_n^4-C\cdot\theta_n)},\quad or\ \Phi_n=x,\ \theta_n=y,\ (n=1,3) \tag{25}$$

$$\therefore\quad x=g(y) \tag{26}$$

$g(x)$ is approximated by Legendre's polynomials P_n over the whole interval $(\alpha_n,\ \beta_n)$ of x.

$$a=3\int_o^1\lambda\cdot g\left\{(\beta_n-\alpha_n)\lambda+\alpha_n\right\}\cdot d\lambda/(\beta_n-\alpha_n)$$

$$b=\int_o^1 g\left\{(\beta_n-\alpha_n)\lambda+\alpha_n\right\}\cdot d\lambda-\alpha_n\cdot a$$

$$\therefore\quad x\fallingdotseq a\cdot y+b \tag{27}$$

$n=1:\ (\alpha_1,\ \beta_1)\qquad \therefore a=a_n,\ b=b_n.$

$n=3:\ (\alpha_3,\ \beta_3)\qquad \therefore a=a'_n,\ b=b'_n.$

From (4) and (18), we must vanish the unknown boundary temperatures $f(z)$ and $\Psi(z)$, and we obtain simultaneous Integro-differential equations of z. To solve the first approximations of these equations, firstly, neglecting small terms and taking linear terms only, these are reduced to simultaneous Volterra's integral equations of z.

In the Volterra's integral equation, we can find the value at $z=0$ exactly and easily, and by the approximation of (27), the only value at $z=0$ is accurate in this case, and so the simultaneous Volterra's integral equations are reduced to the simultaneous algebraic equations of $x_o(0)=\{f(z)/a_n\}_{z=0}$ and $y_o(0)=\{\Psi(z)/a'_n\}_{z=0}$. There is no need to solve the Volterra's integral equation at the whole values of z, but it is sufficient to find the $x_o(0)$ and $y_o(0)$ only, by the method of the fine intervals $(\alpha_n,\ \beta_n)$ and the approximation of (27) as a whole.

$$\therefore\ f_o(0)=x_o(0)-\frac{b_n}{a_n} \tag{28}$$

$$\Psi_o(0)=y_o(0)-\frac{b'_n}{a'_n} \tag{29}$$

We can find the boundary temperatures $f(z=0)$ of (7) and $\Psi(z=0)$ of (21). By the repetitions of the above-mentioned methods, taking some places to find the temperature as the point of $z=0$, the temperatures of all the some parts of the multiple layers of pipes are determined.

3. Numerical Calculations

The author could not find the accurate experimental data on this research and so assumed the values of thermophysical constants for numerical calculations as follows.

The burning gas (temperature $T_g=1200°C$) contains $CO_2=11\%$ ($\varepsilon_{CO_2}=0.135$) and $H_2O=8\%$ ($\varepsilon_{H_2O}=0.19$).

The inside wall with inner radius $r_1=1840$mm is constructed by coating 10 mm thick ($\lambda_1=0.36$kcal/m h °c, $\varepsilon_{w_1}=0.80$), middle wall 150 mm thick is made by fire-proof brick $\{\lambda_2=0.72(1+0.06\ \theta_2)\}$ and outside wall 25 mm thick is made by iron plate $\{\lambda_3=77(1-0.2\theta_3)$, $\varepsilon_{w_3}=0.4\}$. The emissivity of gas $\varepsilon_G=\varepsilon CO_2+C_w\cdot\varepsilon H_2O-\Delta\varepsilon=0.315$.

The angle factor $1/\phi_{GC}=1/\varepsilon_G+(1/\varepsilon_{w_1}-1)$, $\therefore\ \phi_{GC}=0.292$.

Heat transfer coefficient of inside surface $\alpha=10$ kcal/m^2 h °C.

Heat transfer coefficient of outer surface $\alpha=8$ kcal/m^2 h °C.

The air temperature of outside$=30°C$. By these assumptions, K, B and C of (25), J_0, J_1, Y_o and Y_1 of the finite Hankel Transform are calculated. a_n, b_n and a'_n, b'_n of (27) are found at the fine intervals ($\alpha_1=4.70$, $\beta_1=4.65$) and ($\alpha_3=3.27$, $\beta_3=3.25$) which are assumed from the form of (25). The results of numerical calculations : dimensionless gas temperature $=4.87$, dimensionless temperatures of boundary surfaces among θ_1, θ_2 and θ_3 :

$f_o(0)=4.666$, $\Psi_o(0)=4.165$.

The temperature differences at fine intervals : 13°C at (α_1, β_1), 4.5°C at (α_3, β_3).

In the above case for the simplification of numerical calculations, gas temperature and heat transfer coefficient of inner surface were assumed, and so $f_o(0)$ and $\Psi_o(0)$ were found, but when we want to calculate the gas temperature or heat tranfer coefficient of inner surface, if the outer surface temperature is given, we need to reverse the order of calculations or apply the trial methods.

In the case of which the influence of temperature upon ε_{w_1} and ε_{w_3} must be considered, we need to expand $(K-B\theta_n{}^4-C\theta_n)$ of (25) to P_4.

4. Summary

Heat Transfer Problems of cylindrical coordinates are determined by the method of finite Hankel Transforms.

The first point in these analyses is the linearization of non-linear boundary conditions.

The second point is to determine the unknown boundary temperatures.

The simultaneous Integro-differential equations are reduced to the simultaneous Volterra's integral equations, which are reduced to the simultaneous algebraic equations.

We need to solve only the linear algebraic equations by this method of fine interval and approximation as a whole, instead of non-linear Integro-differential equations or non-linear

Fredholm integral equation of the other thermal radiation problems as usual.

In the present paper, one part of the basic researches on cement kiln and centrifugal casting is described.

Nomenclature

$F(Z)$, temperature at $x_1=1$, $(r=r_2)$, [°K]
J_o, Bessel function
J_1, Bessel function
L, length of heating surface, [m]
T_o, atmospheric temperature at the outside of pipe, [°K]
T_g, gas temperature, [°K]
P_n, Legendre's polynomials
Y_o, Neumann function
Y_1, Neumann function
Z, coordinate of axial direction, [m]
a, constant contained radiation constant[2],
a_n, constant
b, constant contained heat transfer coefficient
b_n, constant
a'_n, constant
b'_n, constant
d, constant contained radiation constant[2] and heat transfer coefficient.
d', constant contained radiation constant[2] and heat transfer coefficient.
$f_o(z)=F(Z)/T_o$
$f(z)=a_n \cdot f_o(z)+b_n$
$g(x)$, function of x
$i=1, 2, 3, \cdots$
r, radial length
r_n, radial length $(n=1, 2, 3, 4)$
x, variable
x_n, $=(r+r_n)/(r_{n+1}+r_n)$
y, variable
z, variable$=Z/D_1$
Greek symbols
α_n, constant $(n=1, 3)$
β_n, constant $(n=1, 3)$
εw_1, emissivity
εw_3, emissivity
λ_n, thermal conductivity [kcal/mh °C], $(n=1, 2, 3)$
λ, variable
η, variable
μ, variable
θ_n, $=T_n/T_o$ dimensionless temperature
Φ_n, dimensionless temperature $(n=1, 3)$
$\Psi(Z)$, temperature [°K]

$\Psi(z)$, function of $z=a'_n \cdot \Psi_o(z)+b'_n$

$\Psi_o(z)$, $=\Psi(Z)/T_o$

$\xi_{i,n}$, Eigen-values ($i=1, 2, 3, \cdots$)

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