Transmission of Wave Past a rigid Vertical thin Barrier (2)

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Abstract

The results of an experimental and theoretical investigation on the damping action of submarged vertical thin barrier are presented. The experimental data also are compared with published theories. Furthermore, in this paper, a new theory is presented for the transmission of waves passing through a submarged plane barrier.

This theory is based on a consideration of the Wave maker theory by T. H. Havelook.

Laboratory data are presented to show that these theory are useful for the engineering stand point.

Introduction

One possible type of break water consists of a thin rigid vertical barrier from the sea bottom to some distance below the water surface. One of the problems in the design of such a structure is the determination of the distance above the sea bottom to which such a structure must extend in order to function effectively. A theory of such a structure is developed here together with the results of laboratory tests on a model structure under a variety of wave actions.

Theoretical Development

Three theories are given, one developed herein and two due to, R. A. Fuchs[1] and authors[2].

Fuch's Theory for Power Transmission

In 1951 Fuchs developed a theory for the partical transmission of gravity wave for a submarged vertical thin barrier. He found that

$$K_t = \sqrt{\frac{\sinh 2k_0 h - \sinh 2k_0(h-d) + 2k_0 d}{\sinh 2k_0 h + 2k_0 h}}$$

where k denotes the wave number, h is a still water depth, d denotes a upper edge depth of the submarged vertical barrier, and K_t is a transmission coefficient.

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Momentum Transmission Theory

In 1974 authors developed a theory for the partical transmission and partial reflection of finite amplitude wave in finite depth for a submarged vertical thin barrier. We found that[3]

$$K_t=\sqrt[4]{\frac{\sinh 4k_0h-\sinh 4k_0(h-d)+4k_0d}{\sinh 4k_0h+4k_0h}}, \quad K_r=\sqrt[4]{\frac{\sinh 4k_0(h-d)+4k_0(h-d)}{\sinh 4k_0h+4k_0h}}$$

where K_r denotes a reflection coefficient.

Wave Maker Theory[4),5),6)]

Consider the partial transmission of uniform small amplitude wave in finite depth for a submarged vertical thin barrier.

We consider a two-dimensional motion, in which a vertical plane occupies the line $x=0$, $d\leq y\leq h$, where the axis of y is taken vertically downwards and $y=0$ is the mean surface. (See Fig. 1) The wave motion, being such a could be produced from

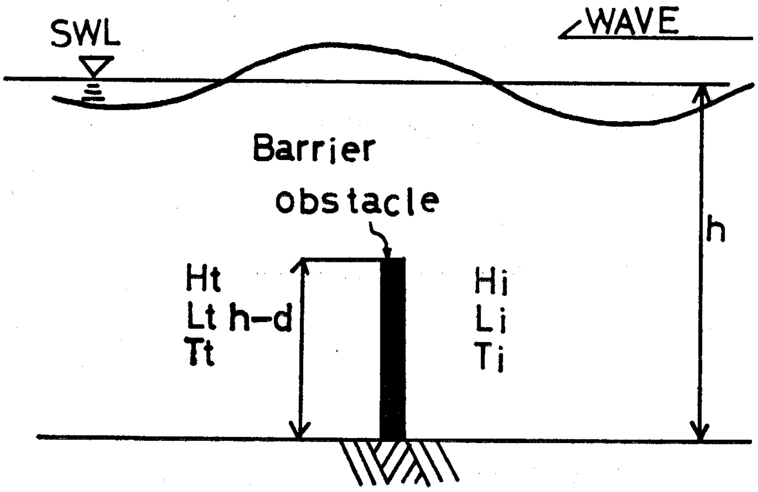


Fig. 1 Definition sketch of submarged obstacle
(H, L, T: Wave Height, Wave Length and Wave Period: Subscript i and t denote are Incident and Transmitted Wave, respectively.)

rest by natural forces, is irrotational and simple harmonic. The fluid is assumed to be incompressible and inviscid so that a velocity potential $\phi(x, y, t)$ exists. The velocity potential satisfies

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0 \tag{1}$$

Neglecting the square of the fluid velocity at the free surface, and omitting the effect of capillarity, the condition at the free surface is

$$\left(\frac{\partial^2 \phi}{\partial t^2} - g\frac{\partial \phi}{\partial y}\right)\Big|_{y=0} = 0 \tag{2}$$

and the surface elevation η is given by

$$\eta = \frac{1}{g}\left(\frac{\partial \phi}{\partial t}\right)\Big|_{y=0} \tag{3}$$

For simple harmonic motion we assume a time factor $e^{i\delta t}$, and equation (2) gives

$$\left(k_0\phi + \frac{\partial \phi}{\partial y}\right)\Big|_{y=0} = 0 \tag{4}$$

Furthermore, the boundary condition on the bottom $y=h$ is that no flow occurs across this surface, i.e.,

$$\frac{\partial \phi}{\partial y}\Big|_{y=h} = 0 \tag{5}$$

The corresponding elementary solutions of equation (1) are

$$\phi = e^{i(\sigma t - k_0 x)} \cosh k_0(y-h) \tag{6}$$

where k_0 is the real positive root of

$$g k_0 \tanh k_0 h = \sigma^2 \tag{7}$$

and

$$\phi_j = e^{i\sigma t - k_j x} \cos k_j(y-h) \tag{8}$$

where k_j is any real positive root of

$$g k_j \tan k_j h + \sigma^2 = 0 \tag{9}$$

This equation has an infinite sequence of real roots, together with an imaginary root ik_0.

In the case of no barrier in the water, equation (6) and (8) give a solution of equation (1). From the linearity of equation (1), also next equation is satisfied equation (1).

$$\Phi = A'\phi + \Sigma B'\phi_j \tag{10}$$

where constant A' and B' are decided from boundary conditions. In the case of existing of barrier in the water, we have the additional boundary condition.

$$\frac{\partial \phi}{\partial x}\Big|_{x=0} = f(y) \sin \sigma t \tag{11}$$

where, we assume then the possibility of expanding a function $f(y)$ in the range $0<y<h$ in the form

$$f(y) = A \cosh k_0(y-h) + \Sigma B_j \cos k_j(y-h) \tag{12}$$

where the summation extends over the real positive roots of the equations (9).

We find that the coefficients are given by

$$A=\frac{4k_0}{2k_0h+\sinh 2k_0h}\int_0^h f(y)\cosh k_0(y-h)dy \tag{13}$$

$$B_j=\frac{4k_j}{2k_jh+\sin 2k_jh}\int_0^h f(y)\cos k_j(y-h)dy \tag{14}$$

From equation (11) and (12), we derived next relation.

$$\left.\frac{\partial\Phi}{\partial x}\right|_{x=0}=\{A\cosh k_0(y-h)+\Sigma B_j\cos k_j(y-h)\}\sin\sigma t \tag{15}$$

The other hand, we obtain next relation from equation (6), (8), (10).

$$\left.\frac{\partial\Phi}{\partial x}\right|_{x=0}=-A'k_0e^{i\sigma t}\cosh k_0(y-h)-\Sigma B'k_je^{i\sigma t}\cos k_j(y-h) \tag{16}$$

From equation (15) and (16), further more we obtain

$$A'=Ak_0^{-1},\quad B'=B_jk_j^{-1} \tag{17}$$

Substituting equation (17) into equation (10), we obtain after development.

$$\begin{aligned}\Phi&=Ak_0^{-1}\cos(\sigma t-k_0x)\cosh k_0(y-h)\\&-\Sigma B_jk_j^{-1}e^{-k_jx}\sin\sigma t\cos k_j(y-h)\end{aligned} \tag{18}$$

Equation (18) gives a velocity potential in the case of existing barrier in the water.

Considering the partial standing wave will arise on the barrier, we use a velosity distribution of partial standing wave for the boundary condition.

$$u|_{x=0}=\left.\frac{\partial\Phi}{\partial x}\right|_{x=0}=-\frac{gTa}{L}\left(1-\frac{a_r}{a}\right)\frac{\cosh k_0(y-h)}{\cosh k_0h}\sin\sigma t \tag{19}$$

where a_r and a denote a reflected wave amplitude and incident wave ampritude, respectively.

From equation (19) and (11), we obtain next relation for $f(y)$

$$d<y<h\qquad f(y)=0 \tag{20}$$

$$0<y<d\qquad f(y)=-\frac{gTa}{L}(1-\sqrt{1-K_t^2})\frac{\cosh k_0(y-h)}{\cosh k_0h} \tag{21}$$

where K_t is a wave transmission coefficient.

The values of A and B_j follow from equation (13) and (14), and from equation (18) we deduce the velocity potential in this case:

$$\begin{aligned}\Phi=&-\frac{gTa(1-\sqrt{1-K_t^2})}{Lk_0\cosh k_0h}\frac{\sinh 2k_0h-\sinh 2k_0(h-d)+2k_0d}{\sinh 2k_0h+2k_0h}\\&\cdot\cosh k_0(y-h)\cos(\sigma t-k_0x)\end{aligned} \tag{22}$$

where we neglect the flow disturbance neighboring the vertical barrier.

Substituting equation (22) into surface conditions we leads to a next relation for surface elevation.

$$\eta=a(1-\sqrt{1-K_t^2})\frac{\sinh 2k_0h-\sinh 2k_0(h-d)+2k_0d}{\sinh 2k_0h+2k_0h}\sin(\sigma t-k_0x) \quad (23)$$

On the other hand, we postulate the transmitted wave form by next relation

$$\eta_t=a_t\sin(\sigma t-k_0x+\varepsilon) \quad (24)$$

Considering the continuous of wave form at $x=0$, we derive the following equation from equation (23) and (24)

$$a_t=a(1-\sqrt{1-K_t^2})\frac{\sinh 2k_0h+2k_0d-\sinh 2k_0(h-d)}{\sinh 2k_0h+2k_0h} \quad (25)$$

where we neglect the phase angle ε.

From equation (25), we derive the next relation for the transmission coefficient Kt.

$$K_t=\frac{2F}{1+F^2},\quad F=\frac{\sinh 2k_0h+2k_0d-\sinh 2k_0(h-d)}{\sinh 2k_0h+2k_0h} \quad (26)$$

In the equation (26), it is a interesting point that function F is coincide with the Fuchs's theory.

Experimental Equipment and Procedure[7),8)]

The experiments were performed using a 0.7 meter wide, 0.9 meter deep and 20 meter long wave test channel at the Yamaguchi University in Ube.

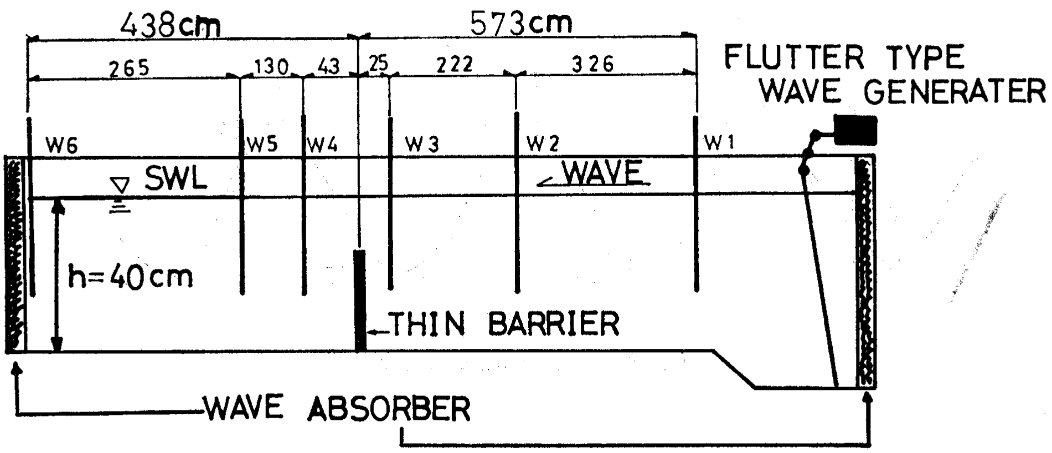


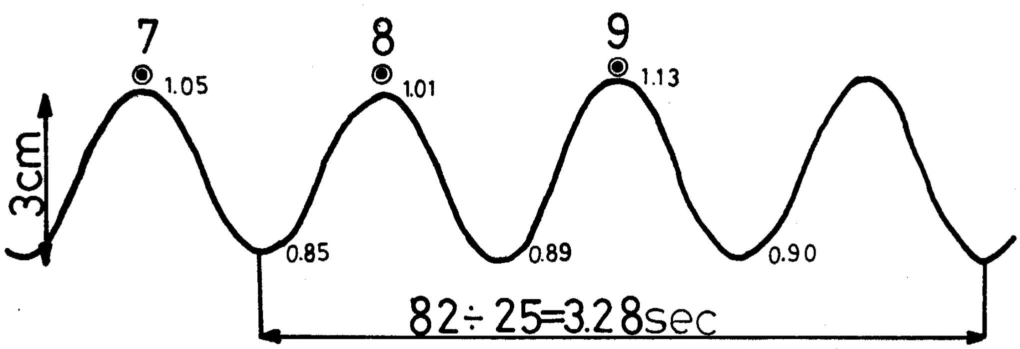
Fig. 2 Arrangement for the Test Channel (wl. . . . w2: Wave gauge).

The rigid vertical thin barrier was fitted into the channel about halfway between the wave generator and the opposite edge. The wave generator was set to give a wave train of given height and period. The wave height were measured with the barrier installed in the desired position. The wave generator was started and measurements were made on waves. The reflected wave height was measured by the Healy's method. Only the data obtained on the lee side wave meter used in computing the transmission coefficient, K_t. In analyzing the data, the first few waves were passed by as these were not of constant period, thereafter the analyzing was made for the next few waves. Considerable care was exercised in placing the barrier so that there would be no leakage between the barrier and the wall of the wave channel.

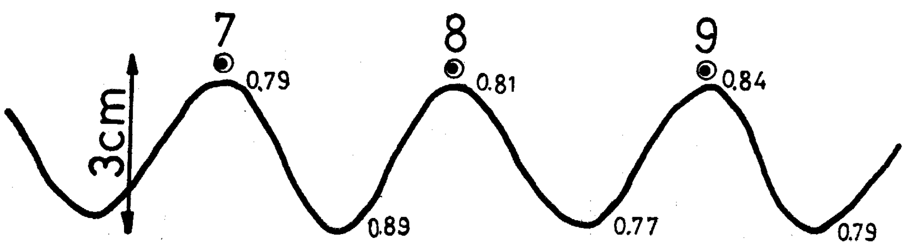
Experimental conditions were as follows Table 1. Fig. 3 shows some examples of measurements.

Table 1. Term of experiments.

h	40.0 (cm)
d	4.0 8.0 12.0 16.0 20.0 24.0 28.0 32.0 (cm)
h/L	0.11 ~ 0.43 (7)
H/L	0.004 ~ 0.099 (32)



Infront of barrier



In lee of barrier

Fig. 3 Sample wave record.

Results

The experimental results are shown in Fig. 4, compared with the wave maker theory, momentum transmission theory and Fuchs's theory.

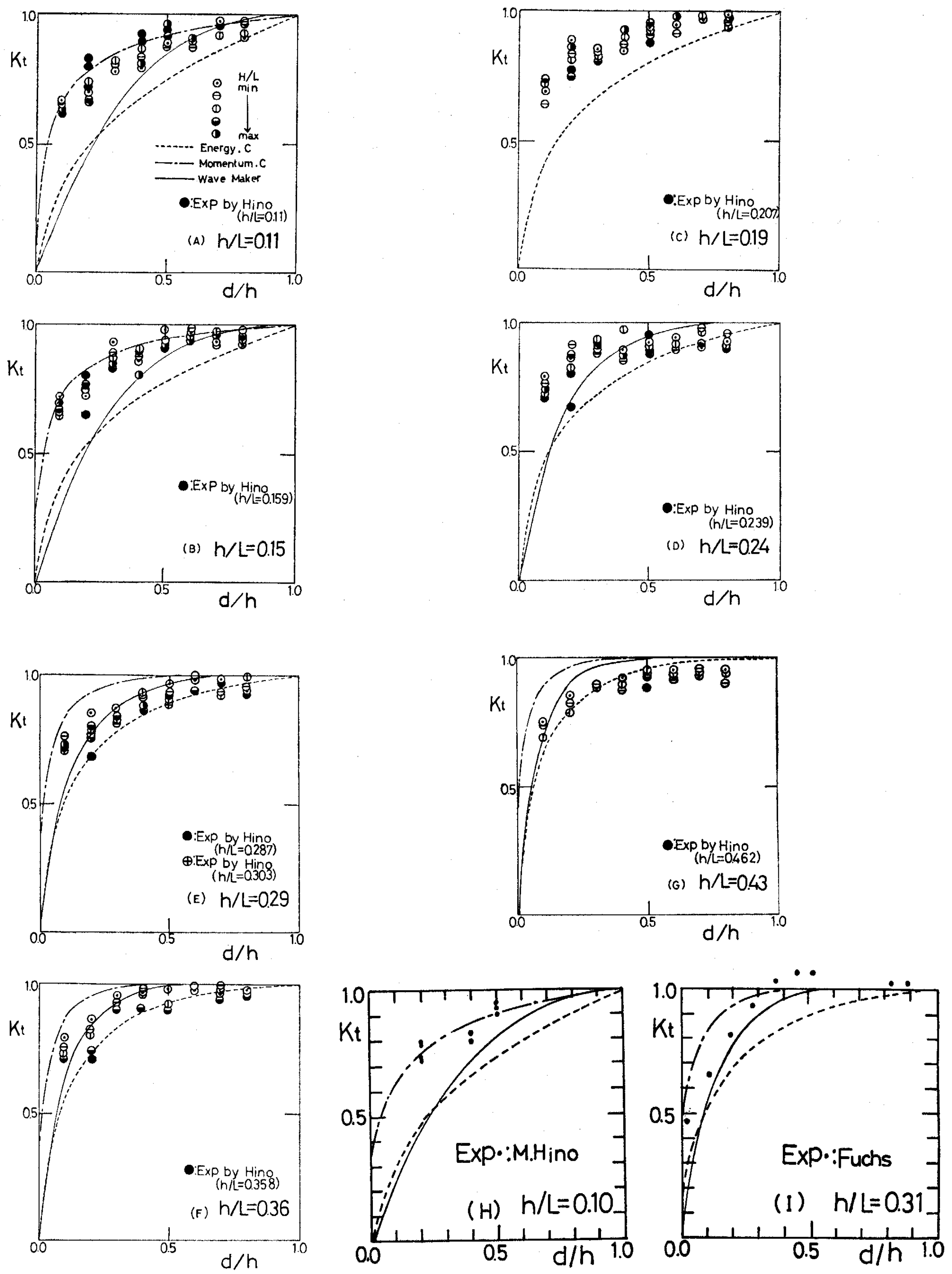


Fig. 4 Comparison of the Experimental Results with Calculated Results.

In Fig. 4, we use jointly with experimental results by T. Hino and Fuchs. It appears that these theory is useful to the engineering, but any improvement in the theory is needed. The trend of increase in the value of the transmission coefficient

K_t with increasing relative depth h/L is consistent. This would be expected from the fact that, all other conditions being equal, the transported wave momentum into lee of barrier increase as the relative depth is increased. The range of long wave, momentum transmission theory predicts the transmission coefficient more closely than does the Fuchs's theory and wave maker theory (equation (26)). Fuchs's theory and wave maker theory predicts the transmission coefficient closely for shallow water wave, but Fuchs's theory predicts the smaller value of transmission coefficient than does the wave maker theory. These would be expected from the experimental results by T. Hino and Fuchs.

Conclusion

The wave maker theory predicts the transmission coefficient adequately for shallow water wave, and the momentum transmission theory does closely for long wave. Fuchs's theory predicts the smaller value than does the others. But these theory is useful to some engineering standpoint. A consistent trend of increasing transmission coefficient with increasing relative depth is evident in the laboratory measurement. This would be expected from the by T. Hino and Fuchs.

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