Particle-Crushing Properties of Sands under High Stresses

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Abstract

A method is proposed for evaluating the particle-crushing properties of granular materials under shear stresses. The increase of specific surface area, ΔS, is used as a measure of the amount of particle-crushing instead of the conventional measure such as particle breakage B.

Drained shear tests are performed on two kinds of sands, and the relation between ΔS and W is examined, where W is the plastic work per unit volume of a sample during shear. It is found that there exists a unique relation between ΔS and W, irrespective of the magnitude of confining pressures.

The ratio dS/dW is defined as an indication of particle-crushing properties of a granular material under shear stresses, in the light of Tanaka's comminution law. It is found that the value of dS/dW has a direct relation with the dilatancy rate $dv/d\varepsilon_1$. Thus, the particle-crushing properties is successfully related to the shear characteristics of granular materials.

Introduction

In a few studies on the high pressure behaviors of granular materials, a significant fact has been observed that the angle of shearing resistance ϕ_d of a granular soil in a dense state is rapidly decreased with increase of the confining pressures. The pressure range where ϕ_d is rapidly decreased seems to depend on the particle strength of the material. Therefore, it is presumed that the change of the angle of shearing resistance has a certain relation with the particle-crushing phenomenon.

For making clear the effect of particle-crushing on the shear characteristics of a soil, it is necessary to evaluate the amount of particle-crushing. As a measure of the particle-crushing, Marsal[1] used the particle breakage B and investigated the relation between the value of B and the shear characteristics of rockfill materials at elevated confining pressures. The value of B is determined from the change of grain size distribution of a material before and after the shear test. However, the physical meaning of the particle breakage B is somewhat obscure and this measure may be invalid for comparing the amount of particle-crushing of two materials of different initial gradings.

In this paper, the authors adopt the increase of specific surface area as a measure of the amount of particle-crushing, and study the effect of particle-crushing on the drained shear characteristics of sands under high confining pressures up to 200 kg/cm^2.

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Sample and Test Procedure

The samples used in this study are quartz sand and feldspar sand, which were prepared by crushing the quartzite and feldspar in massive states. The ground materials were sieved into the three kinds of different gradations, of which various properties and gradings are shown in Table 1 and Fig. 1, respectively. Dry samples were compacted with a tamper in a rubber membrane of 1 mm thick, and specimens of initial void ratio of about 0.62 were made in a size of 50 mm diameter and 125 mm height. In order to decrease the end friction of specimen, a couple of lubricated end platens[2] were used instead of the conventional rough end platens. The platens consist of cylinder 56 mm in diameter by 40 mm thick with 10 mm diameter porous disc at the center for providing drainage. Each platen was covered with a thin coat of silicon grease and separated from the specimen by a circular rubber membrane, 0.2 mm in thickness, with a hole in the center.

Drained shear tests at constant cell pressures were carried out by using a high pressure triaxial compression apparatus. The details of test procedure and apparatus have been described elsewhere[3].

Table 1. Various properties of samples.

Sample	Specific gravity G_s	Maximum diameter D_{max} (mm)	Diameter of 50% size D_{50} (mm)	Uniformity coefficient U_c	Maximum void ratio e_{max}	Minimum void ratio e_{min}
Quartz sand A	2.64	0.59	0.28	3	1.40	0.60
〃 B	〃	〃	0.24	8	1.22	0.49
〃 C	〃	〃	0.19	15	1.13	0.43
Feldspar sand A	2.57	0.59	0.29	3	1.41	0.61
〃 B	〃	〃	0.23	11	1.23	0.50
〃 C	〃	〃	0.19	16	1.13	0.44

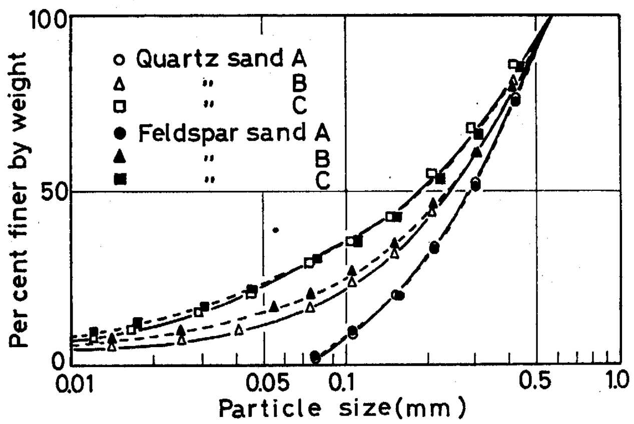

Fig. 1 Particle-size distribution curve of original sample.

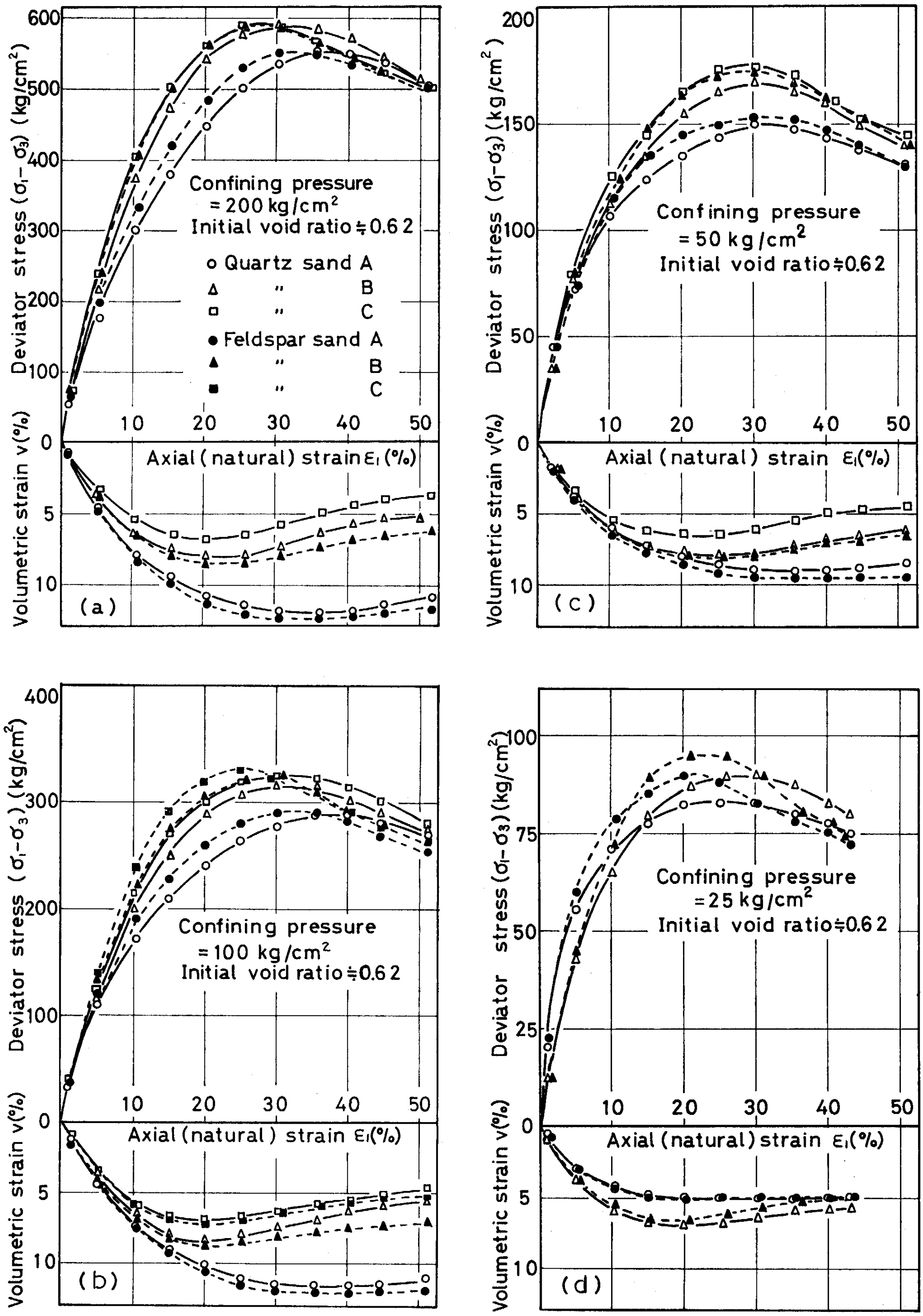

Fig. 2 Stress-strain curve on the sample in drained shear test.

Shear Characteristics

Drained shear tests were performed at various confining pressures up to 200 kg/cm^2, and the typical stress-strain curves were as shown in Fig. 2. By comparing the stress-strain curves of the two kinds of sands of the same initial gradings, it is seen that the shear strength of the quartz sand is slightly lower than that of the feldspar sand. On the other hand, at the time of failure, the tendency to dilate, that is, the dilatancy effect, of the quartz sand is larger than that of the feldspar sand. The variations of the maximum principal stress ratio $(\sigma'_1/\sigma'_3)_f$, and the dilatancy rate at failure $(dv/d\varepsilon_1)_f$, with increase of the confining pressures were as illustrated in Fig. 3.

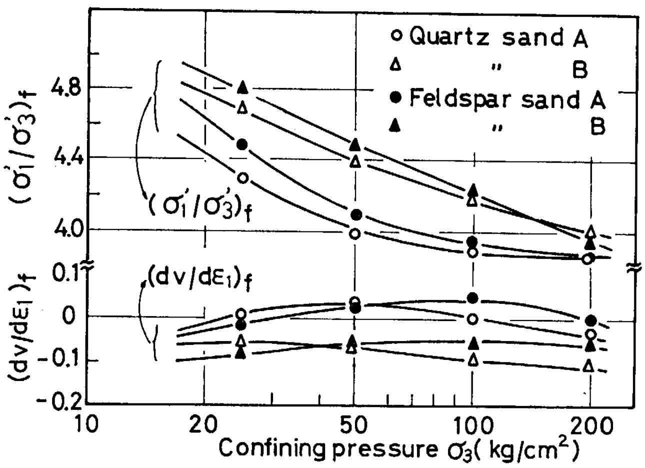

Fig. 3 Relation between confining pressure σ_3 and dilatancy rate $(dv/d\varepsilon_1)_f$ or principal stress ratio $(\sigma'_1/\sigma'_3)_f$, at failure.

The differences of the shear characteristics of the two sands may be discussed by the Rowe's stress-dilatancy equation[4],

$$\frac{\sigma'_1}{\sigma'_3}=\left(1-\frac{dv}{d\varepsilon_1}\right)\tan^2\left(45+\frac{\phi_\mu}{2}\right) \tag{1}$$

where, ϕ_μ is the particle-to-particle friction angle. The values of ϕ_μ of the two sands were not measured, in this study, but they are estimated that $\phi_\mu=26°$ for quartz sand[4] and $\phi_\mu=37°$ for feldspar sand[5]. Of course these values of ϕ_μ are valid only when the relative movement of particles in contact is caused by sliding, not by rolling or crushing. Therefore, it cannot be said that the difference of the values of ϕ_μ is a main cause of the difference of shear strengths between the two sands. Nevertheless, the difference of ϕ_μ might contribute, partly, to the difference of shear strengths of the two sands, because the value of $(\sigma'_1/\sigma'_3)_f$ of the feldspar sand is larger than that of the quartz sand in spite of the dilatancy effect of the former being smaller than that of the latter (Fig. 3 and 4).

The effect of initial grading on the shear characteristics of the sand is illustrated in Fig. 4, in which the grading property is represented by uniformity coefficient U_c. It is seen that the maximum principal stress ratio and the dilatancy effect increase as the uniformity coefficient increases. These results probably suggest that the shear characteristics of granular materials under high pressures are significantly affected by the particle-crushing properties of the samples. The problem how the particle-crushing affects the shear characteristics of sample will be discussed in the following chapters.

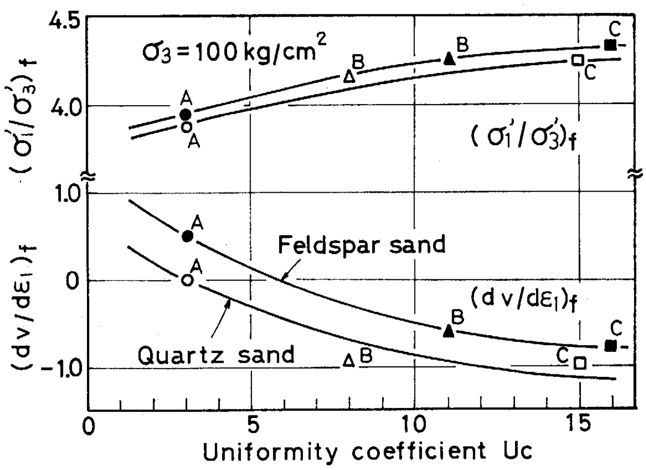

Fig. 4 Variation of the principal stress ratio $(\sigma_1'/\sigma_3')_f$ or dilatancy rate $(dv/d\varepsilon_1)_f$, at failure, with increase of the uniformity coefficient of original sample U_c.

Change of Amount of Particle-Crushing during Shear

As a measure of the amount of particle-crushing, the authors have adopted the increase of specific surface area ΔS[6]. The specific surface area of the fraction of a sample finer than 74 micron was measured by Blaine method, which is widely used for measuring the specific surface area of cement powder. As to the fraction coarser than 74 micron, the specific surface area was calculated from the mean diameter of each sample remained on sieves in sieve analysis. The values obtained by these two methods were summed up and the specific surface area of the sample was calculated.

An example of the change of specific surface area of a quartz sand with increase of the axial strain (in natural) was as shown in Fig. 5. Similar relations have been obtained on the Toyoura sand[7] and Shirasu[8], and those are also shown in the figure. It is noted in this figure that the value of ΔS continues to increase even after the stress increment becomes zero or negative, that is, ΔS is a function of axial strain ε_1. It is axiomatical that ΔS is also a function of stress σ, thus,

$$\Delta S = f(\sigma, \varepsilon_1) \tag{2}$$

This equation leads to a hypothesis that the amount of particle-crushing expressed by ΔS is a function of work done by an external force acting on the sample.

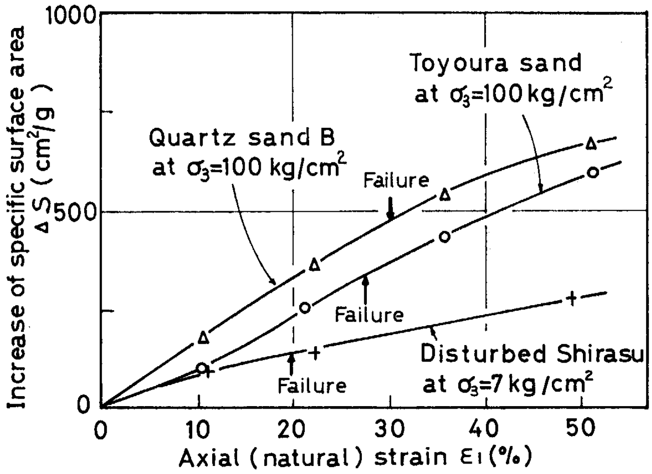

Fig. 5 Relation between increase of specific surface area ΔS and axial strain ε_1.

Particle-Crushing Properties of Sands during Shear

On the sample under deformation, the total energy change per unit volume $\delta E'$ has been divided by Roscoe et al[9] into the two components, namely, the internal stored and recoverable, elastic, energy δU and the energy dissipated in shear or frictional heat loss δW.

$$\delta E' = \delta W + \delta U \tag{3}$$

For the triaxial compression test, the right-hand side of eq. (3) is rewritten as the function of the effective principal stresses, σ'_1, σ'_3, and the principal strain increments, $\delta\varepsilon_1$, $\delta\varepsilon_3$, as follows:

$$\left.\begin{aligned} \delta E' &= \sigma'_1 \delta\varepsilon_1 + 2\sigma'_3 \delta\varepsilon_3 \\ &= q\delta\varepsilon + p\delta v \\ &= (q\delta\varepsilon + p\delta v)_e + (q\delta\varepsilon + p\delta v)_s \end{aligned}\right\} \tag{4}$$

where,

$$q = \sigma'_1 - \sigma'_3$$

$$p = \frac{1}{3}(\sigma'_1 + 2\sigma'_3)$$

$$\delta v = \delta\varepsilon_1 + 2\delta\varepsilon_3$$

$$\delta\varepsilon = \delta\varepsilon_1 - \frac{\delta v}{3}$$

and subscript e or s refers to recoverable elastic component or non-recoverable plastic component, respectively. On the basis of the Rittinger's idea[10] that the increase of specific surface area is proportional to the work done, the relation between ΔS and work was examined. In this case, it may be proper to adopt the plastic component of work, W, as the particle-crushing is apparently a non-recoverable phenomenon.

From the eqs. (3) and (4),

$$W = \int(q\delta\varepsilon + p\delta v) - \int(q\delta\varepsilon + p\delta v)_e$$

but, introducing the Roscoe's hypothesis[9] $(q\delta\varepsilon)_e = 0$,

$$W = \int(q\delta\varepsilon + p\delta v) - \int(p\delta v)_e \tag{5}$$

Each term of the right-hand side of the above equation can be determined by the measurement of area under each stress-strain curve.

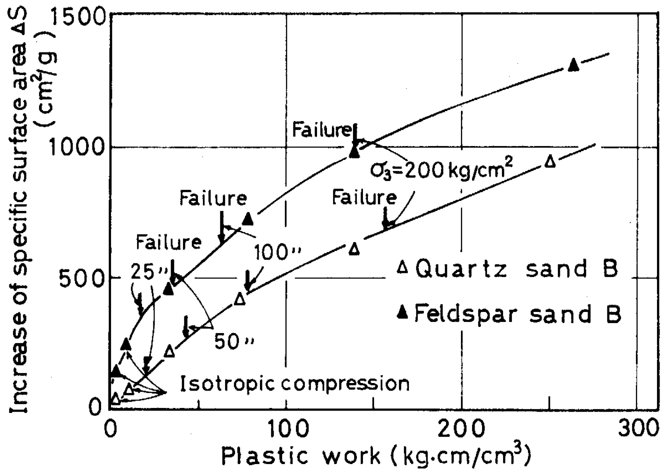

Fig. 6 Relation between increase of specific surface area ΔS and plastic work W on the quartz sand B and feldspar sand B during shear.

The relation curves between ΔS and W are shown in Fig. 6. It is noted in this figure that there is a unique line through the points obtained by a series of drained shear tests on a sand at various confining pressures. This indicates that the amount of particle-crushing during shear is closely related with the plastic work per unit volume of sample. However, the aspect of this curve is somewhat different from the Rittinger's linear relation. It seems that $\Delta S - W$ curve is well explained by Tanaka's equation rather than by Rittinger's. By considering the limitation of comminution, Tanaka[11] has presented an equation in the form:

$$dS/dE = K(S_\infty - S) \tag{6}$$

where, S_∞ is the final value of specific surface area of particles, K is the comminution coefficient, and dS/dE is the energy-efficiency. The ratio dS/dW, which is given by the slope of the $\Delta S - W$ curve, may be relevant to dS/dE, and this ratio indicates the rate of the energy dissipated in the particle-crushing to the total plastic energy. The larger the rate of energy dissipated in particle-crushing to the total plastic energy is, the steeper the slope of the corresponding point on the curve becomes.

It is considered that the particle-crushing directly affects the volume change of a granular material under shear stresses. Hence, it is expected that the particle-crushing property defined by dS/dW has a certain relation with the dilatancy charactertistics such as $(dv/d\varepsilon_1)$. The relation between $(dv/d\varepsilon_1)_f$, and $(dS/dW)_f$, at the time of failure, is represented in Fig. 7, which shows that there exists a direct relation between the two indications.

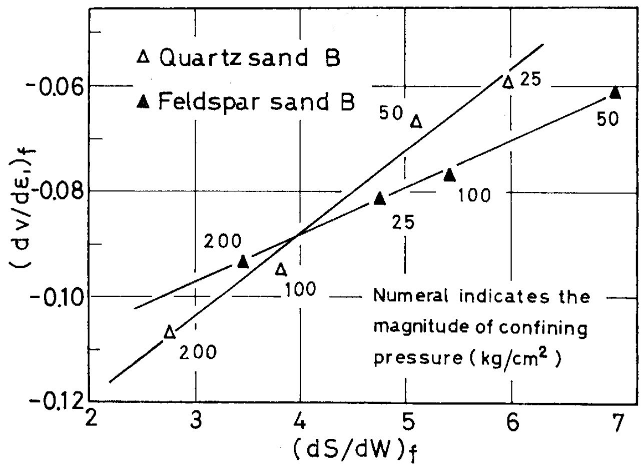

Fig. 7 Relation between the value of $(dS/dW)_f$ and the dilatancy rate $(dv/d\varepsilon_1)_f$, at failure, on the quartz sand B and feldspar sand B.

Conclusions

For making clear the effect of particle-crushing on the shear characteristics of granular materials, high pressure triaxial tests were performed on quartz sands and feldspar sands of various initial gradings. It was shown that the increase of specific surface area ΔS is a valid measure of the amount of particle-crushing, compared with the conventional measure such as particle breakage B. It was found that the amount of particle-crushing continues to increase even after the stress increment becomes zero or negative. It was thereby suggested that the amount of particle-crushing of a granular material under shear stresses may be a function of work done.

The plastic work per unit volume of a sample W was determined by the energy equation and the relation between ΔS and W was examined, showing that this relation of any sample can be expressed by a unique curve irrespective of the magnitude of confining pressures. It was considered that the slope of this curve dS/dW is an excellent indication of particle-crushing property of a granular material, in the light of the comminution law proposed by Rittinger or Tanaka.

It was found that the value of $(dS/dW)_f$ is directly related with the corresponding dilatancy rate $(dv/d\varepsilon_1)_f$. Thus, it could be successfully explained the effect of particle-crushing on the shear characteristics of granular materials.

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