Transmission of Wave Past a rigid Vertical Thin Barrier

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Abstract

A theory is presented for the transmission of waves passing through a rigid vertical thin barrier extending from above the water surface to some distance below the surface.

This theory is based on consideration of WAVE MOMENTUM transmission. Laboratory data are presented to show that this theory is useful for the engineering design standpoint.

Introduction

One possible type of breakwater consists of a thin rigid barrier extending from above the water surface to some distance below the water surface. This barrier may be sidingly mounted on a pile structure, for example. One of the problems in the design of such a structure is to determine the distance below the free surface, to which such a barrier must extend in order to function effectively.

A theory of such a structure is developed here together with the results of laboratory tests on a model structure under a variety of wave conditions.

Theoretical Development[1),2),3),4)]

Three theories are given, one developed herein and two due to F. Ursell and R.L. Wiegel.

Ursell's Theory for Deep Water

In 1947 Ursell developed a theory for the partial transmission and partial reflection of gravity water waves in deep water for a fixed vertical infinitely thin barrier extending from the water surface to some depth below the surface.

He found that

$$K_T=\frac{H_T}{H_I}=\frac{K_1(2\pi d/L)}{\sqrt{\pi^2 I_1^2(2\pi d/L)+K_1^2(2\pi d/L)}} \qquad \cdots\cdots\cdots(1)$$

where K_1 and I_I denote the modified Besell's Function, L is wave length, K_T is transmission coefficient, H_I and H_T are incident wave height and transmitted wave height respectively.

Wiegel's Theory for Power Transmission

In 1960 Wiegel developed a theory for the partial transmission and partial reflection of uniform long-crested periodic water waves in finite depth for a fixed vertical infinitely thin barrier extending from the water surface to some depth below the surface.

He found that

$$K_T = \frac{H_T}{H_I} = \sqrt{\frac{\sinh 2k(h-d)+2k(h-d)}{\sinh 2kh+2kh}} \qquad \cdots\cdots\cdots(2)$$

where k denotes the wave number, h is water depth, d is the submerged length of vertical thin barrier.

Momentum Transmission Theory[5),6)]

Consider the partial transmission and partial reflection of uniform finite amplitude shallow water waves in finite depth for a fixed vertical infinitely thin barrie rextending from the water surface to some depth below the surface.

The wave motion, being such as could be produced from rest by natural forces, is irrotational.

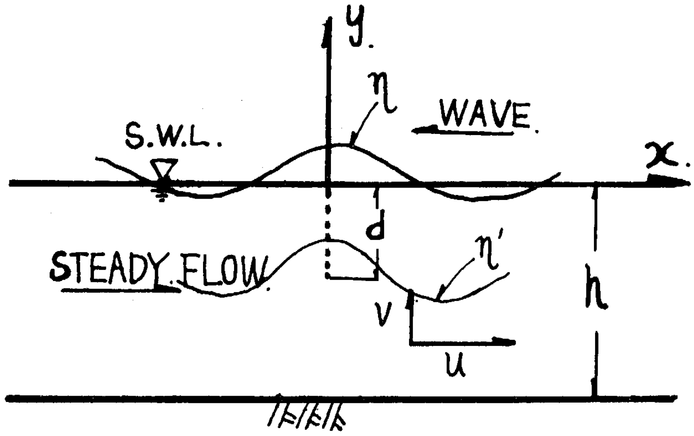


Fig. 1 Definition sketch for the finite amplitude wave.

The fluid is assumed to be incompressible.

Laplace's equation therefore is expressed by

$$\nabla^2\phi=0 \qquad \cdots\cdots\cdots(3)$$

or

$$\nabla^2\psi=0 \qquad \cdots\cdots\cdots(4)$$

where ϕ and ψ denote the velocity potential and stream function respectively. The boundary condition on the bottom, $y=-h$, is that no flow occurs across this surface, i.e.,

$$V=\frac{\partial\phi}{\partial y}=-\frac{\partial\psi}{\partial x}=0 \qquad y=-h \qquad \cdots\cdots\cdots(5)$$

From Eqs. (3), (4) and (5), we assume that ϕ and ψ are given by the following equations.

$$\phi=cx-c\beta\cosh k(h+y)\sin kx \qquad \cdots\cdots\cdots(6)$$

$$\psi=cy-c\beta\sinh k(h+y)\cos kx \qquad \cdots\cdots\cdots(7)$$

Using the relation

$$(\psi)_{y=\eta}=0 \qquad \cdots\cdots\cdots(8)$$

we lead an equation:

$$\eta=\beta \sinh k(h+\eta)\cos kx \qquad \cdots\cdots\cdots(9)$$

from which we obtain the following equation:

$$\eta=\frac{1}{2}ka^2 \coth kh+a \cos kx+\frac{1}{2}ka^2 \coth kh \cos 2kx$$

$$+\frac{1}{4}k^2a^3\left(\coth^2 kh+\frac{1}{2}\right)\cos 3kx+ \qquad \cdots\cdots\cdots(10)$$

Hence, considering the progressive wave, we have

$$\eta=\frac{1}{2}ka^2 \coth kh+a \cos k(x-ct)+\frac{1}{2}ka^2 \coth kh \cos 2k(x-ct)$$

$$+\frac{1}{4}k^2a^3\left(\coth^2 kh+\frac{1}{2}\right)\cos 3k(x-ct), \qquad \cdots\cdots\cdots(11)$$

where $$a=\beta \sinh kh\left\{1+\frac{3}{4}k^2\beta^2\left(1+\frac{3}{4}\sinh^2 kh\right)\right\}. \qquad \cdots\cdots\cdots(12)$$

In a similar manner, we obtain a relation for .
by using the relation

$$(\psi)_{y=\eta'-d}=cd, \qquad \cdots\cdots\cdots(13)$$

we derive the following equation:

$$\eta'=\frac{1}{2}kr^2 \coth k(h-d)+r \cos kx+\frac{1}{2}kr^2 \coth k(h-d) \cos 2kx$$

$$+\frac{1}{4}k^2r^3\left\{\coth^2 k(h-d)+\frac{1}{2}\right\} \cos 3kx, \qquad \cdots\cdots\cdots(14)$$

where $$r=a[\sinh\{k(h-d)\}/\sinh kh]. \qquad \cdots\cdots\cdots(15)$$

The momentum of uniform flow between the water surface and some level d located below the wave trough is given by

$$M_u=\rho cdL+\frac{1}{2}ka^2\rho cdL\left\{\coth kh-\frac{\sinh k(h-d) \cosh k(h-d)}{\sinh^2 kh}\right\} \cdots\cdots\cdots(16)$$

On the other hand, the momentum of stationary wave between $\psi=0$ and $\psi=cd$ is given by

$$M_{sw}=-\int_0^L\int_{\psi=cd}^{\psi=0}\rho\frac{\partial\psi}{\partial y}dxdy=\rho cdL \qquad \cdots\cdots\cdots(17)$$

We find finally the wave momentum between $\psi=0$ and $\psi=cd$ as follow:

$$M=\pi\rho a^2\, c\left\{\coth kh-\frac{\sinh k(h-d)\cosh k(h-d)}{\sinh^2 kh}\right\} \quad \cdots\cdots\cdots(18)$$

from which, we obtain the momentum distribution:

$$m'=\frac{\pi\rho c k a^2}{\sinh^2 kh}\cosh 2k(h-d) \quad \cdots\cdots\cdots(19)$$

Further, from Eqs. (18) the mass transport velocity u is determined as follows:

$$u'=\frac{1}{\rho L}\frac{\partial M}{\partial d}=ck^2\, a^2\,\frac{\cosh 2k(h-d)}{2\sinh^2 kh} \quad \cdots\cdots\cdots(20)$$

Momentum distribution is shown in Fig. 2.

Fig. 3 shows the energy distribution.

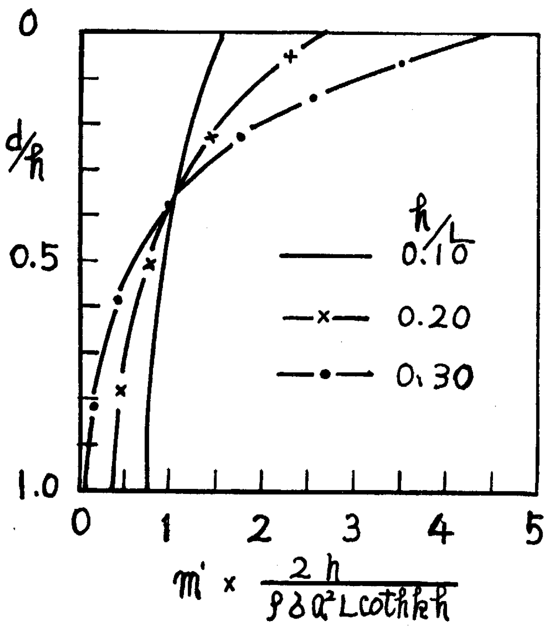


Fig. 2 Distribution of the wave momentum.

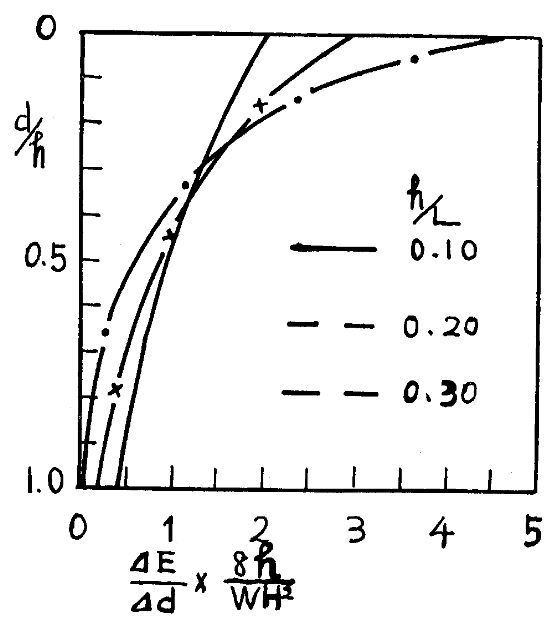


Fig. 3 Distribution of the wave energy.

Case 1. Surface Obstacle

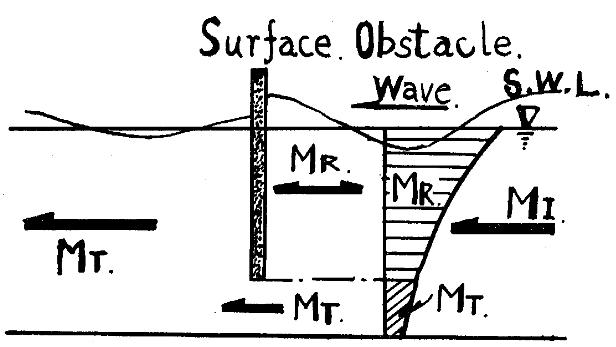


Fig. 4 Definition sketch for the assumption.

The following assumption have been made in working out the theory of wave transmission and reflection by a surface obstacle.

1: Incident wave is finite amplitude wave.

2: The only momentum transmitted is that from the bottom to the depth d; that is, the momentum between the water surface and the depth d is reflected (see Fig. 4).

3: Water is homogeneous, and external forces such as those due to the earth rotation and the lunar and solar attractions are neglected, because of the comparatively small body of water contained in the channel. With these assumptions, a theory of momentum transmission and reflection by a surface obstacle can be developed as follow:

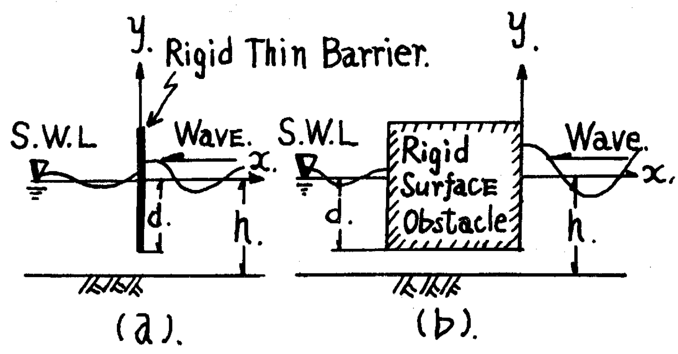


Fig. 5 Definition sketch of the surface obstacle.

From Eqs. (19) and (20), we obtain

$$M_I = \frac{\rho c^2 k^2 H_I^4}{256 \sinh^4 kh} (\sinh 4kh + 4kh) \qquad \cdots\cdots\cdots(21)$$

$$M_R = \frac{\rho c^2 k^2 H_R^4}{256 \sinh^4 kh} (\sinh 4kh + 4kh)$$

$$= \frac{\rho c^2 k^2 H_I^4}{256 \sinh^4 kh} \{\sinh 4kh - \sinh 4k(h-d) + 4kd\} \qquad \cdots\cdots\cdots(22)$$

$$M_T = \frac{\rho c^2 k^2 H_T^4}{256 \sinh^4 kh} (\sinh 4kh + 4kh)$$

$$= \frac{\rho c^2 k^2 H_I^4}{256 \sinh^4 kh} \{\sinh 4k(h-d) + 4k(h-d)\}\,, \qquad \cdots\cdots\cdots(23)$$

where M_I denotes the transported momentum of incident wave over the whole depth, M_R the wave momentum reflected between the water surface and the depth d below the still water level, and M_T the wave momentum transmitted between the bottom and the depth d below the still water level. Considering the conservation of momentum, we obtain

$$M_I = M_T + M_R + f M_I \qquad \cdots\cdots\cdots(24)$$

where f is the coefficient of momentum loss.

Substituting Eqs. (21), (22) and (23) into Eqs. (24), we obtain, after development.

$$(1-f)=\left(\frac{H_T}{H_I}\right)^4+\left(\frac{H_R}{H_I}\right)^4=K_T^4+K_R^4, \qquad \cdots\cdots\cdots(25)$$

where K_T and K_R denote the transmission coefficient and reflection coefficient respectively.

The portion of wave momentum transmitted between the bottom and the depth d below the still water level is

$$\frac{M_T}{M_I}=\frac{\dfrac{\rho c^2k^2H_I^4}{256\sinh^4 kh}\{\sinh 4k(h-d)+4k(h-d)\}}{\dfrac{\rho c^2k^2H_I^4}{256\sinh^4 kh}(\sinh 4kh+4kh)}, \qquad \cdots\cdots\cdots(26)$$

and the portion of wave momentum reflected between the water surface and the depth d below the still water level is

$$\frac{M_R}{M_I}=\frac{\dfrac{\rho c^2k^2H_I^4}{256\sinh^4 kh}\{\sinh 4kh-\sinh 4k(h-d)+4kd\}}{\dfrac{\rho c^2k^2H_I^4}{256\sinh^4 kh}(\sinh 4kh+4kh)}, \qquad \cdots\cdots\cdots(27)$$

the portion of the wave height transmitted between the bottom and the depth d below the still water level is

$$\frac{H_T}{H_I}=\sqrt[4]{\frac{M_T}{M_I}}=\sqrt[4]{\frac{\sinh 4k(h-d)+4k(h-d)}{\sinh 4kh+4kh}}. \qquad \cdots\cdots\cdots(28)$$

This function is shown in Fig. 8. This shows that the ratio of the transmitted to incident wave heights is proportional to the fourth power of the ratio of the transmitted to incident wave momentum flux. Similarly, the portion of wave height reflected between the water surface and the depth d below the still water level is

$$\frac{H_R}{H_I}=\sqrt[4]{\frac{M_R}{M_I}}=\sqrt[4]{\frac{\sinh 4kh-\sinh 4k(h-d)+4kd}{\sinh 4kh+4kh}}, \qquad \cdots\cdots\cdots(29)$$

which shows that the ratio of the reflected to incident wave height is proportional to the fourth power of the ratio of the reflected to incident wave momentum flux.

Case 2. Submerged Obstacle

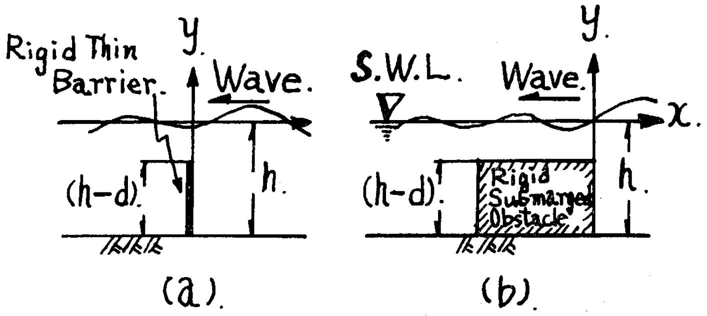


Fig. 6 Definition sketch of the submarged obstacle.

The preceding assumptions are also made in working out the theory of wave transmission and reflection by a submerged obstacle. The theory in this case, can be developed as follow:

From Eqs. (19) and (20), we obtain

$$M_I=\frac{\rho c^2k^2H_I^4}{256\sinh^4kh}(\sinh 4kh+4kh) \qquad \cdots\cdots\cdots(30)$$

$$M_R=\frac{\rho c^2k^2H_I^4}{256\sinh^4kh}\{\sinh 4k(h-d)+4k(h-d)\} \qquad \cdots\cdots\cdots(31)$$

$$M_T=\frac{\rho c^2k^2H_I^4}{256\sinh^4kh}\{\sinh 4kh-\sinh 4k(h-d)+4hd\}\,. \qquad \cdots\cdots\cdots(32)$$

In a similar manner, the portion of surface wave momentum transmitted between the water surface and the depth d below the still water level is

$$\frac{M_T}{M_I}=\frac{\sinh 4kh-\sinh 4k(h-d)+4kd}{\sinh 4kh+4kh}\,, \qquad \cdots\cdots\cdots(33)$$

and the portion of wave momentum reflected between the bottom and the depth d below the still water level is

$$\frac{M_R}{M_I}=\frac{\sinh 4k(h-d)+4k(h-d)}{\sinh 4kh+4kh} \qquad \cdots\cdots\cdots(34)$$

The portion of the wave height transmitted between the water surface and the depth d below the still water level is given by,

$$\frac{H_T}{H_I}=\sqrt[4]{\frac{M_T}{M_I}}=\sqrt[4]{\frac{\sinh 4kh-\sinh 4k(h-d)+4kd}{\sinh 4kh+4kh}}\,, \qquad \cdots\cdots\cdots(35)$$

which shows that the ratio of the transmitted to incident wave height is proportional to the fourth power of the ratio of the transmitted to incident wave momentum flux.

Similarly the portion of the wave height reflected between the bottom and the depth d below the still water level is given by

$$\frac{H_R}{H_I}=\sqrt[4]{\frac{M_R}{M_I}}=\sqrt[4]{\frac{\sinh 4k(h-d)+4k(h-d)}{\sinh 4kh+4kh}}, \quad \cdots\cdots\cdots(36)$$

which shows that the ratio of the reflected to incident wave heights is proportional to the fourth power of the ratio of the reflected to incident wave momentum flux.

Experimental Equipment and Procedure

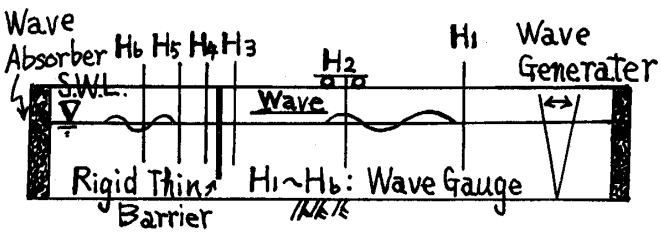


Fig. 7 Arrangement for the test channel.

The experiments were performed using a 0.7 meter wide 0.9 meter deep and 20 meter long wave test channel at the university of Yamaguchi at Tokiwa.

The rigid vertical thin barrier was fitted into the channel about halfway between the wave generator and the opposite edge. The wave generator was set to give a wave train of given height and period. The wave height were measured with the barrier installed in the desired position.

The wave generator was started and measurements were made on waves. The reflected wave height was measured by the Healy's method. Only the data obtained on the lee side wave meter were used in computing the transmission coefficient, H_T/H_I.

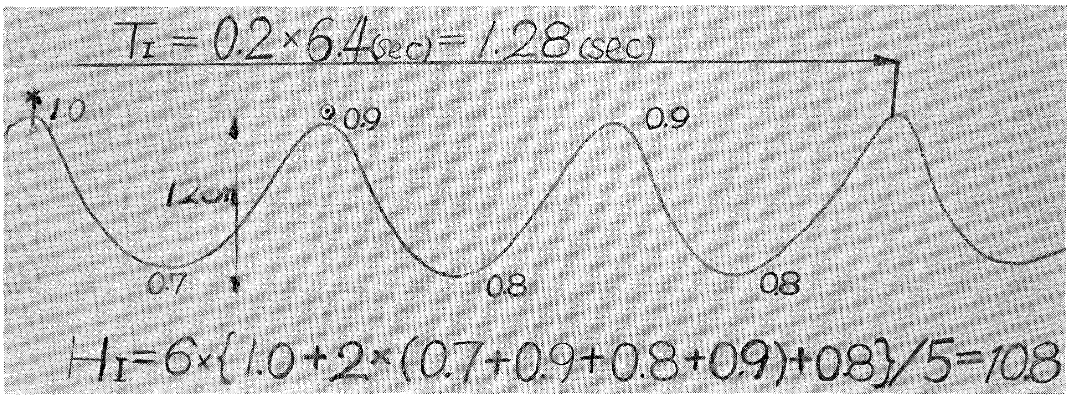
In analyzing the data, the first few waves were passed by as these were not of constant period, thereafter the analyzing was made for the next few waves.

Considerable care was exercised in placing the barrier so that there would be no leakage between the barrier and the walls of the wave channel. Experimental conditions were as follows:

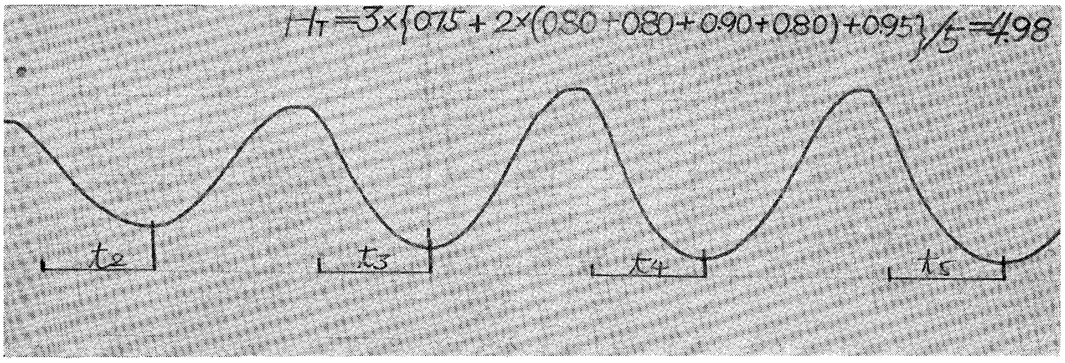
Fig. 8 shows examples of measurements taken in the channel.

Table 1. Experimental conditions.

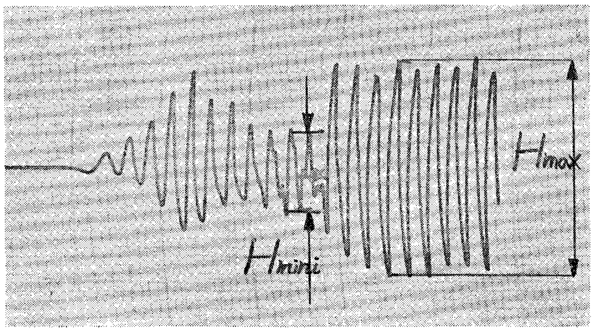
Water Depth	25, 30, 35, 40 (cm)
h-d	1, 5, 10, 15, 20 (cm)
H_1/L	0.005~0.1
h/L	0.08~0.5



Infront of barrier



In lee of Barrier



Reflected wave

Fig. 8 Sample wave record.

Results

The results are shown in Fig. 9∼17, compared with the momentum transmission theory.

It appears that the theory is useful to the engineering, but any improvement in the theory is needed. The trend of decrease in the value of the transmission coefficient H_T/H_I with increasing wave steepness is consistent. This would be expected from the fact that, all other conditions being equal, the wave momentum increases as the wave steepness is increased, hence the momentum loss due to separation at the bottom of the barrier would increase with increasing wave steepness, resulting in decrease in the

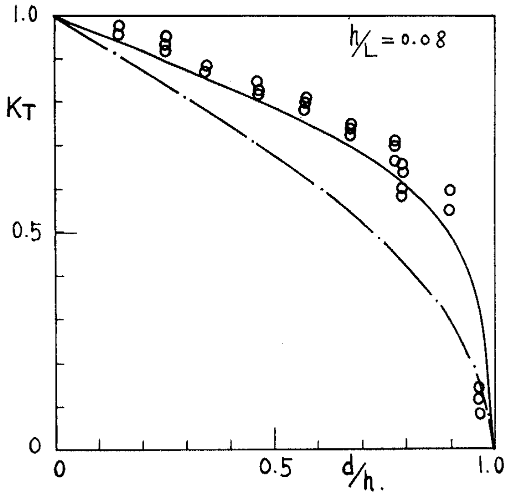


Fig. 9

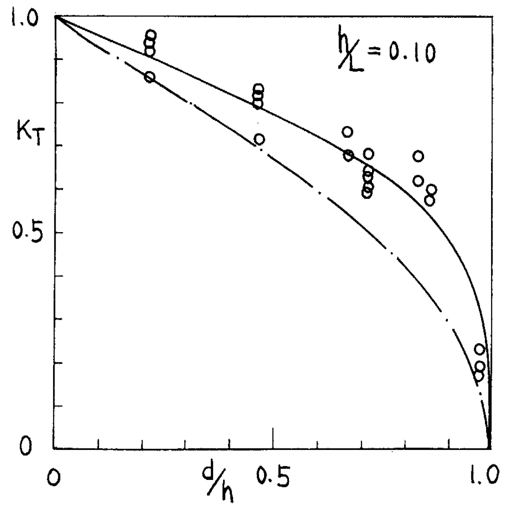


Fig. 10

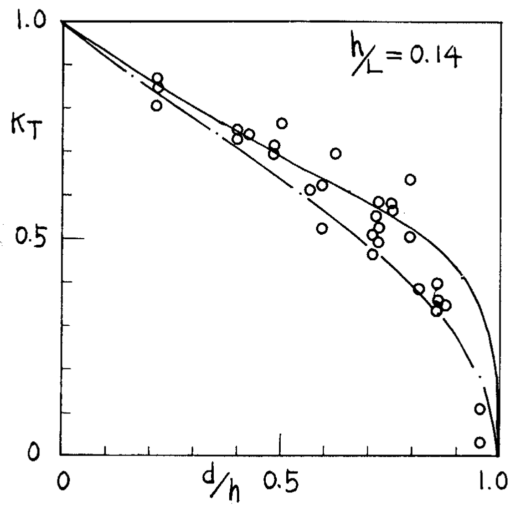


Fig. 11

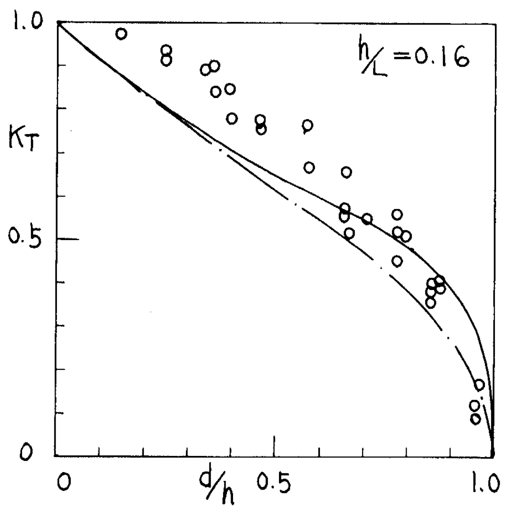


Fig. 12

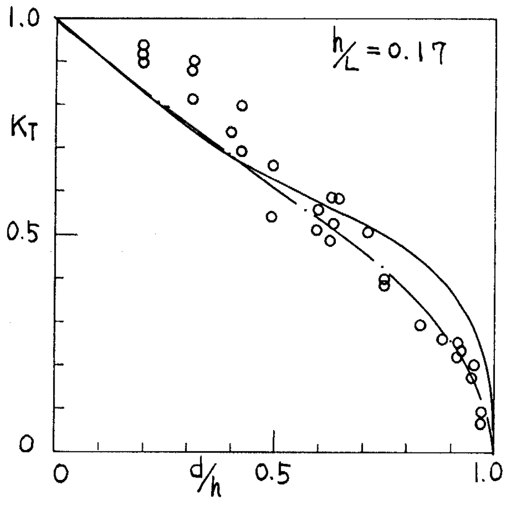


Fig. 13

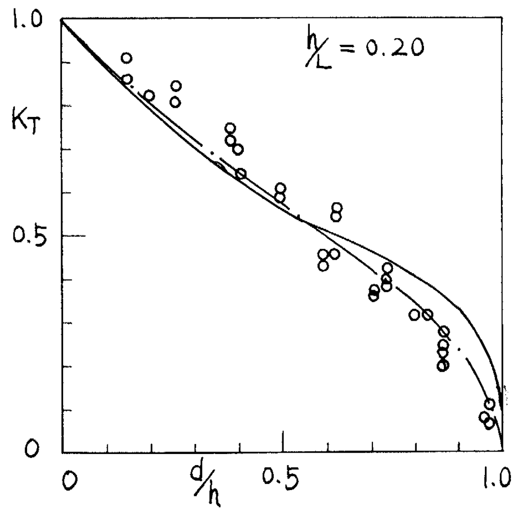


Fig. 14

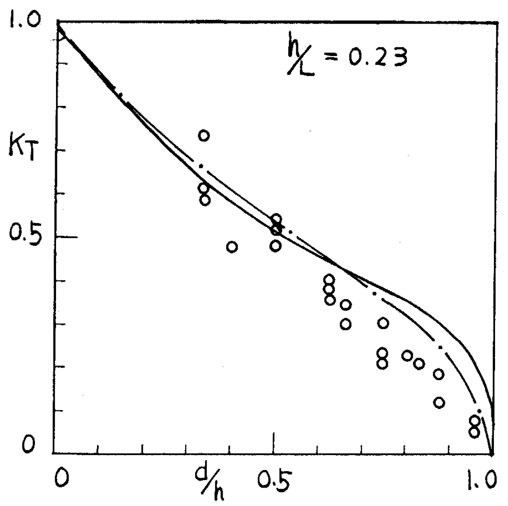


Fig. 15

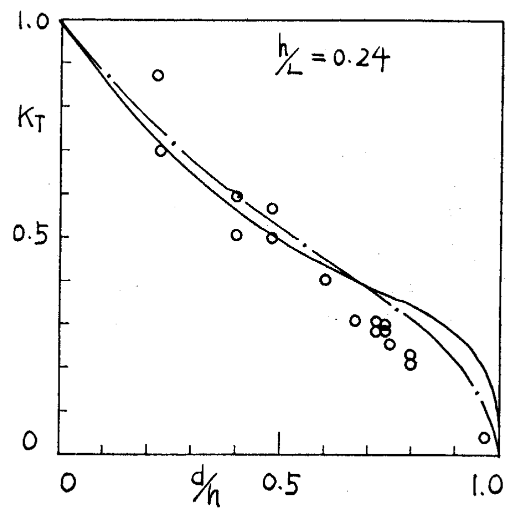


Fig. 16

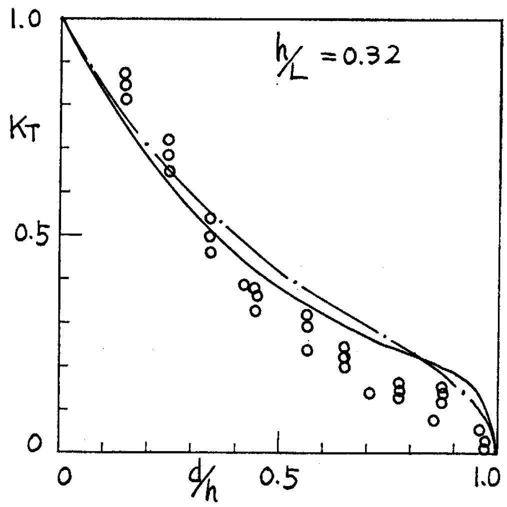


Fig. 17

Fig. 9–17 Comparison of the experimental results with calculated results.
(○ Exp, — Author, —·— Wiegel)

transmission coefficient. The momentum transmission theory predicts the transmission coefficient more closely for long waves than does the power transmission theory.

Conclusion

The momentum transmission theory predicts the transmission coefficient adequately for some engineering purpose. A consistent trend of decreasing transmission coefficient with increasing wave steepness is evident in the laboratory measurements. The portion of the transmitted wave height to incident wave height is proportional to the fourth power of the ratio of the transmitted wave momentum to the incident wave momentum.

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