Effect of the Electrical Conductivity of the Walls on the Induction-Heating of the Electrically Conducting Fluid

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Abstract

The temperature distribution of the inductively heated fluid is investigated for the idealized channel-model. The model is composed of the infinite plane channel and the semi-infinite current sheet. Two infinite plates with electrical conductivity are placed a distance $2L$ apart and the conducting fluid flows through the channel with only the x component of the velocity. The electromagnetic field with high frequency is supplied by the semi-infinite sheets on the plates.

The normalized temperature for that model is obtained by using Maxwell's equations, Ohm's law and the energy equation. As a result, the effect of the conductivity of the walls is shown by the factor S. At the high-frequency field this factor is expressed by exp $(-\gamma_w l)$ approximately, where the parameter γ_w is defined by $2L/\delta_w$ and l by h/L (h : thickness) and δ_w is the skin depth of the wall.

The radiation losses and the dependency of the temperature on the fluid properties are not considered in this analysis.

1. INTRODUCTION

It has been shown experimentally in a few works[1)~4)] that the temperature distribution of the induction-coupled plasma indicates the off-axis peak. The theoretical analysis for the characteristic was done in a previous paper[5)]. There the electrical conductivity of the wall has not been considered.

When conductive materials are used as wall substances in order to realize higher energy sources[6)], the effect of wall conductance on the temperature of the inductively heated fluid cannot be ignored and then some modifications must be done.

The purpose of this work is to study the effect of the wall conductance on the channel flow. The analysis is performed for the idealized model which is shown in Fig. 1. The system is

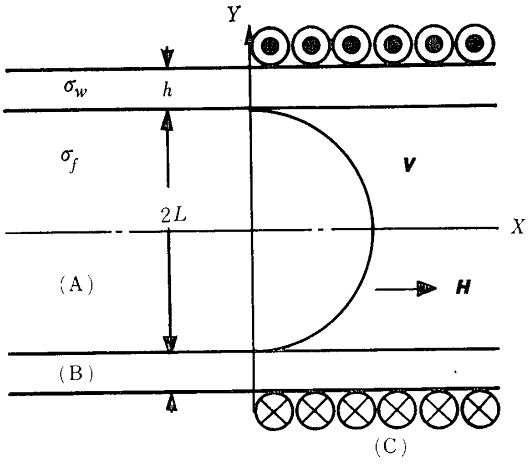

Fig. 1 The geometry of an idealized model for infintite plane channel. (A) : conducting fluid, (B) : conducting wall, (C) : current sheet.

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composed of the infinite plane channel and the semi-infinite current sheet which produces the electromagnetic field with high frequency. The infinite plates form xz planes and the high-frequency current passes through the sheet in the z direction. These assumptions allow the flow to be uncoupled from electromagnetic equations. The temperature of the fluid can be obtained from the energy equation under some boundary conditions ; the energy equivalent to Joule heating is drived from Maxwell's equations and Ohm's law.

The radiation losses and the dependency of the temperature on the fluid properties may be important in our analysis but they are not considered. This is a preliminary report in order to perform more detailed research.

2. BASIC EQUATIONS AND SOLUTIONS FOR UNIFORM AND PARABOLIC FLOWS

The important assumptions in this analysis are summarized as follows ;

(1) Physical properties are constant and uniform for the fluid and the wall respectively.
(2) The temperature at the surface of the outer wall is constant and equal to Θ_0.
(3) The heat conduction in the x direction is neglected in comparison with the radial conduction.
(4) The viscous dissipation and the radiation losses are neglected in comparison with thermal conductance.
(5) The displacement current and the space charge are small enough to be neglected.
(6) The end effects of the field are ignored.

Under these assumptions the energy equations for the fluid and the wall may be solved.

The physical quantities are expressed as follows ;

velocity : $\boldsymbol{V}=(u(y),\ 0,0)$,
magnetic field : $\boldsymbol{H}=(H(y)e^{i\omega t},\ 0,0)$,
current density : $\boldsymbol{J}=(0,0,J(y))$,
electric field : $\boldsymbol{E}=(0,0,E(y))$,
temperature : $\Theta=\Theta(x,\ y)$.

The energy equations for the fluid with the steady convective flow and for the wall are given by

$$\rho\ c_p\ u\frac{\partial\Theta}{\partial x}=k\frac{\partial^2\Theta}{\partial y^2}+\frac{J_f{}^2}{\sigma_f},\tag{1}$$

$$k\frac{\partial^2\Theta}{\partial y^2}+\frac{J_w{}^2}{\sigma_w}=0\ .\tag{2}$$

Here ρ is mass density, σ electrical conductivity, k thermal conductivity, c_p specific heat, J current density. The suffix f and w denote the values for the fluid and the wall respectively. The term (J^2/σ), which means Joule heat, may be drived from Maxwell's equations and Ohm's law and they are represented as follows ;

$$\text{rot}\ \boldsymbol{E}=-\mu_0\frac{\partial\boldsymbol{H}}{\partial t},\tag{3}$$

$$\text{rot}\ \boldsymbol{H}=\boldsymbol{J},\tag{4}$$

$$\text{div}\ \boldsymbol{E}=0,\tag{5}$$

$$\text{div}\ \boldsymbol{H}=0,\tag{6}$$

$$\boldsymbol{J}=\sigma\boldsymbol{E},\tag{7}$$

where the constant μ_0 dinotes the permeability in a vacuum. From these equations, the induction equations can be drived and written as

$$\frac{\partial^2 H}{\partial y^2}=\frac{2i\ H}{\delta_f{}^2},\tag{8}$$

$$\frac{\partial^2 H}{\partial y^2} = \frac{2iH}{\delta_w^2}, \tag{9}$$

where both $\delta_f=(2/\mu_0\sigma_f\omega)^{1/2}$ and $\delta_w=(2/\mu_0\sigma_w\omega)^{1/2}$ are called skin depth.

It is assumed that magnetic field H becomes H_0 at the surface of the outer wall and that magnetic field is continuous at the boundary surface between the fluid and the wall. An appropriate solutions for magnetic field is

$$H=\frac{H_0\cosh\left(\frac{1+i}{\delta_w}L\right)\cdot\cosh\left(\frac{1+i}{\delta_f}y\right)}{\cosh\left(\frac{1+i}{\delta_w}(L+h)\right)\cdot\cosh\left(\frac{1+i}{\delta_w}L\right)} \qquad \text{for } |y|\leq L, \tag{10}$$

$$H=\frac{H_0\cosh\left(\frac{1+i}{\delta_w}y\right)}{\cosh\left(\frac{1+i}{\delta_w}(L+h)\right)} \qquad \text{for } L\leq|y|\leq L+h. \tag{11}$$

The expressions of the current density J are obtained from Eq. (4). These are

$$J_f=-\frac{H_0(1+i)\cosh\left(\frac{1+i}{\delta_w}L\right)\cdot\sinh\left(\frac{1+i}{\delta_f}y\right)}{\delta_f\cosh\left(\frac{1+i}{\delta_w}(L+h)\right)\cdot\cosh\left(\frac{1+i}{\delta_f}L\right)} \qquad \text{for } |y|\leq L, \tag{12}$$

$$J_w=-\frac{H_0\ (1+i)\ \sinh\left(\frac{1+i}{\delta_w}y\right)}{\delta_w\cosh\left(\frac{1+i}{\delta_w}(L+h)\right)} \qquad \text{for } L\leq|y|\leq L+h. \tag{13}$$

Therefore the energy equivalent to Joule heat is given by

$$\frac{J_f^2}{\sigma_f} = \frac{2H_0^2}{\sigma_f\delta_f^2}\cdot g_f(\eta), \tag{14}$$

$$\frac{J_w^2}{\sigma_w} = \frac{2H_0^2}{\sigma_w\ \delta_w^2}\cdot g_w(\eta). \tag{15}$$

In these equations, η and $g(\eta)$ are defined by

$$\eta = \frac{y}{L}, \tag{16}$$

$$g_f(\eta)=\frac{\cosh\gamma_w+\cos\gamma_w}{\cosh(\gamma_w(1+l))+\cos(\gamma_w(1+l))}\cdot\frac{\cosh(\gamma_f\eta)-\cos(\gamma_f\eta)}{\cosh\gamma_f+\cos\gamma_f}, \tag{17}$$

$$g_w(\eta)=\frac{\cosh(\gamma_w\eta)-\cos(\gamma_w\eta)}{\cosh(\gamma_w(1+l))+\cos(\gamma_w(1+l))}, \tag{18}$$

where $\gamma_w=\frac{2L}{\delta_w}$, $\gamma_f=\frac{2L}{\delta_f}$ and $l=\frac{h}{L}$ (thickness ratio).

If we use the quantities defined by

$$\theta = \frac{2k\sigma_f}{H_0^2}(\Theta-\Theta_0),\quad u=u_m f(\eta),\quad u_m=\frac{1}{2L}\int_{-L}^{L}u\,dy,$$

$$Pe' = \frac{\rho\,c_p\,u_m\,L}{k} \text{ (Pe'clet Number)},$$

$$\xi = \frac{x}{L} \text{ and } \chi = \frac{\xi}{Pe'}=\frac{x}{Pe'L}, \tag{19}$$

the energy equations for the fluid and the wall can be written as

$$f(\eta)\frac{\partial\theta}{\partial\chi} = \frac{\partial^2\theta}{\partial\eta^2}+\gamma_f^2 g_f(\eta), \tag{20}$$

$$\frac{\partial^2\theta}{\partial\eta^2}+\left(\frac{\sigma_f}{\sigma_w}\right)\gamma_w^2 g_w(\eta) = 0. \tag{21}$$

The boundary condition for the wall (Eq.(21)) is

$$\theta=0 \qquad \text{for } \eta=\pm(1+l). \tag{22}$$

And we obtain as the solution

$$\theta = \left(\frac{\sigma_f}{\sigma_w}\right)\left(1-\frac{\cosh(\gamma_w\eta)+\cos(\gamma_w\eta)}{\cosh(\gamma_w(1+l))+\cos(\gamma_w(1+l))}\right). \tag{23}$$

The temperature at the inner surface of the wall becomes

$$\theta_B = \left(\frac{\sigma_f}{\sigma_w}\right)\left(1-\frac{\cosh\gamma_w+\cos\gamma_w}{\cosh(\gamma_w(1+l))+\cos(\gamma_w(1+l))}\right). \tag{24}$$

Since, at the contact surface between the fluid and the wall, the wall temperature is equal to the fluid temperature, Eq.(20) will be solved. When we introduce a new variable of temperature

$$T=\theta-\theta_B, \tag{25}$$

Eq.(20) and the boundary conditions have the following forms

$$f(\eta)\frac{\partial T}{\partial \chi} = \frac{\partial^2 T}{\partial \eta^2}+\gamma_f^2 g_f(\eta), \tag{26}$$

$$T = 0 \qquad \text{for } \chi=0, \tag{27a}$$

$$T = 0 \qquad \text{for } \eta=\pm 1, \tag{27b}$$

$$\frac{\partial T}{\partial \eta}= 0 \qquad \text{for } \eta=0. \tag{27c}$$

At infinity downstream the heat flux in the x direction is assumed to be neglected in comparison with that in the y direction. Thus there we have the temperature T_∞ determined by the following equation.

$$\frac{d^2T}{d\eta^2}+\gamma_f^2 g_f(\eta)=0. \tag{28}$$

An appropriate solution is

$$T_\infty = S\left(1-\frac{\cosh(\gamma_f\eta)+\cos(\gamma_f\eta)}{\cosh\gamma_f+\cos\gamma_f}\right), \tag{29}$$

where $$S=\frac{\cosh\gamma_w+\cos\gamma_w}{\cosh(\gamma_w(1+l))+\cos(\gamma_w(1+l))}. \tag{30}$$

When the variable $\phi=T-T_\infty$ is introduced here, Eqs.(26) and (27) are simplified in the following forms

$$f(\eta)\frac{\partial\phi}{\partial\chi} = \frac{\partial^2\phi}{\partial\eta^2}, \tag{31}$$

$$\phi=-T_\infty \quad \text{for } \chi=0, \tag{32a}$$

$$\phi=0 \qquad \text{for } \eta=\pm 1, \tag{32b}$$

$$\frac{\partial\phi}{\partial\eta}=0 \qquad \text{for } \eta=0. \tag{32c}$$

This differential equation may be solved by using the solutions which were obtained in the previous paper[5].

The solution for the uniform flow is given by

$$T=T_\infty+\phi$$
$$= S\left\{1-\frac{\cosh(\gamma_f\eta)+\cos(\gamma_f\eta)}{\cosh\gamma_f+\cos\gamma_f}+\sum_{n=1}^{\infty}A_n\exp(-\omega_n^2\chi)\cos(\omega_n\eta)\right\}, \tag{33}$$

where $$A_n=(-1)^{n-1}\left\{\frac{2\omega_n}{\alpha}\cdot\frac{\beta\gamma_f^2-\alpha\omega_n^2}{\gamma_f^4-\omega_n^4}-\frac{2}{\omega_n}\right\},$$

$$\alpha=\cosh\gamma_f+\cos\gamma_f,\quad \beta=\cosh\gamma_f-\cos\gamma_f,$$

$$\omega_n=\left(n-\frac{1}{2}\right)\pi.$$

The solution for the parabolic flow is also given by

$$T= S\left\{1-\frac{\cosh(\gamma_f\eta)+\cos(\gamma_f\eta)}{\cosh\gamma_f+\cos\gamma_f}+\sum_{n=1}^{\infty}B_n\exp(-\lambda_n\chi)\Phi_n(\eta)\right\}, \tag{34}$$

where

$$B_n=\frac{1}{C_n}\int_{-1}^{1}\left(1-\frac{\cosh(\gamma_f\eta)+\cos(\gamma_f\eta)}{\cosh\gamma_f+\cos\gamma_f}\right)\Phi_n(\eta)(1-\eta^2)d\eta,$$

$$\Phi_n(\eta)=\exp\left(-\frac{1}{2}\mu_n\eta^2\right)\left\{1+\sum_{m=1}^{\infty}\frac{(1-\mu_n)(5-\mu_n)\cdots(4m-3-\mu_n)}{(2m)!}\mu_n^m\ \eta^{2m}\right\},$$

$$C_n=\int_{-1}^{1}(\Phi_n(\eta))^2(1-\eta^2)d\eta,$$

$$\mu_n=\left(\frac{3}{2}\lambda_n\right)^{1/2}.$$

In Eqs. (33) and (34), the parts of the curly brackets become the solutions[5] which have been obtained without cosidering the wall conductance. Thus the effect of the wall is included in the factor S which is determined by the parameters of γ_w and l.

3. NUMERICAL RESULTS

Numerical calculations will be limited in the case when the fully ionized gases can be treated as the conducting fluid. The electrical conductivity for fully ionized gases is assumed to obey the theoretical equation given by Spitzer and Härm[7]. The conductivity is $1\sim50\Omega^{-1}\text{cm}^{-1}$ for temperature T$\sim$10000°K and number density of electron Ne$\sim10^{16}$ in the case of Argon. The approximately same values are experimentally obtained for the induction-coupled Argon plasmas at atmospheric pressure[3)8)]. The conductivity of copper and iron are about $10^6\Omega^{-1}\text{cm}^{-1}$ and $10^5\Omega^{-1}\text{cm}^{-1}$ respectively.

If the frequency of the electromagnetic field is 1 MHz and half depth of the channel is 5 cm, skin depth become 1$\sim$10 cm for fully ionized gases and $10^{-2}\sim10^{-3}$ cm for the wall (copper or iron). The parameter γ_f becomes 1$\sim$10 and γ_w $10^3\sim10^4$.

In the high-frequency field the parameter γ_w has the very large value. Therefore in this case the factor S is given by the approximate expression ;

$$S\sim\exp(-\gamma_w l),$$
$$=\exp(-\sqrt{2\mu_0}\cdot\sqrt{\sigma_w}\cdot\sqrt{\omega}\cdot h). \qquad (35)$$

The part of the exponent is composed of the product of the wall thickness, the square root of the wall conductivity and the square root of the frequency. When the value 3×10^{-3}(h=0.15mm)

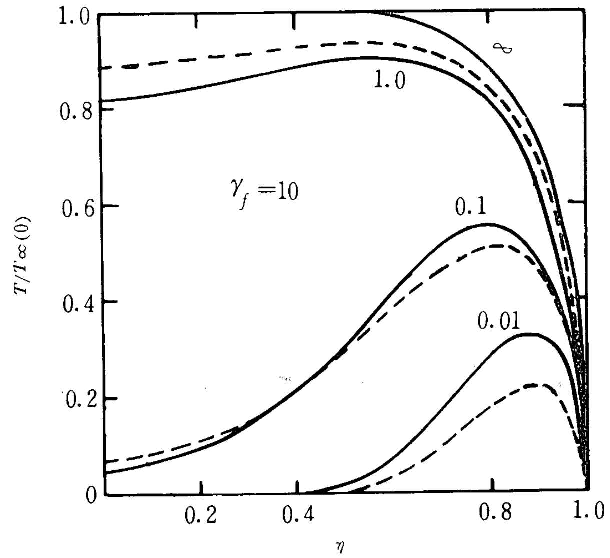

Fig. 2 The temperature profile of the plane channel for $\gamma_f=10$. The dashed curves correspond to uniform flow and the full curves to parabolic flow. The numbers next to the curves give the values of $\chi(=x/(Pe'L))$.

is used for the thickness ratio, the modification factor S and thus temperature of the fluid become so small as to be a few tenths.

The normalized temperature$(T/T_{\infty}(0))$ is calculated for $\gamma_f=10$ and shown in Fig. 2. The dashed curves correspond to the uniform flow and the solid curves to the parabolic flow. The number belonging to the curves gives the value of χ. For small value of χ the temperature distribution have an off-axis peak as having been shown in experimental works[1)~4)]. With the increase of χ the location of the peak moves towards the channel axis and at infinity downstream the distribution becomes monotonic. The temperature is shown to be higher near the wall in the case of the parabolic flow than the uniform flow but the situation is reversed in the vicinity of the axis.

4. CONCLUSION

The temperature of the conducting fluid which is inductively heated is effected extremly by wall conductance and wall thickness. In conclusion, the influence is shown by the modification factor S, which has approximately the form of exp $(-\gamma_w l)$ in the case of the high-frequency field. It is expected that the same idea developed here is applied to an analysis of the ring discharge which is composed of the two zones apparently ; the layers of high and low conductivity.

If we choose the fully ionized gases as the conducting fluid, the situation will be complicated. In this case the radiation losses and the dependency of the temperature on the electrical and the thermal conductivity become serious problems. Some analytical studies of the induction-heating in the presence of these effects are now in progress.

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