Structure of the Pulsatile Blood Flow in the Vascular with a Rough Wall

Suketsugu NAKANISHI* and Hideo OSAKA**

(Received July 14, 1988)

Abstract

The blood flow in the vascular with a rough wall has been investigated numerically to examine the influence of the roughness on the vascular lesion from the hydrodynamical point of view. As a two−dimensional model of the blood flow in the vascular with a rough surface, the pulsatile flow in the rigid channel with several roughness elements which consist of a series of square ribs is considered, and the blood is assumed to be a Newtonian fluid. The Navier−Stokes equations (stream function−vorticity formulation) are solved by the finite difference method. The calculation is performed for the range of Reynolds number $300 \leqq Re \leqq 2000$, but both the period and the amplitude of oscillation are fixed. In this paper, particular attension is paid to the flow pattern over the roughness elements, so the instantaneous stream lines of pulsatile flow are presented for the neighborhood of roughness elements. The influence of roughness on the vascular lesion is discussed from the structure of pulsatile flow over a rough wall.

1. Introduction

The origin of vascular lesion has not been conclusively elucidated so far, in spite of many theoretical and experimental works.[1)~7)] The present series of numerical studies are aimed at examining the influence of roughness on the vascular lesion from hydrodynamical point of view. In the previous paper,[8), 9)] the authors have numerically studied the steady flow of a viscous fluid through the two−dimensional channel with several roughness elements as a model of the blood flow in an artery with rough surface. The pulsatile flow model is more suitable than the steady flow model for describing the phenomena concerned with the blood flow through a blood vessel in vivo. Therefore, the pulsatile flow of a viscous fluid through a two−dimensional channel with roughness elements was investigated numerically in this paper, and was examined to predict the shearing stress variation on the wall with the roughness elements and the oscillating flow pattern. The oscillating flow over the roughness is characterized by the five non−dimensional parameters : the two geometrical paramerers K/H and W/H (K, W and H are the roughness height, the space of roughness elements and the width of channel, respectively) and the flow parameters, the Reynolds number Re, the Womersley number α and the ratio of flow rate Q_1/Q_0

*Department of physics, Hiroshima Institute of Technology, Itsukaichi, Saeki−ku, Hiroshima, 731−51

**Department of Mechanical Engineering, Yamaguchi University, Tokiwadai, Ube, 755

(Q_1 and Q_0 are the amplitude of oscillating flow rate and the mean flow rate, respectively) . Since the number of parameters involved in this study is so large, we can choose one particular channel geometry, $K/H=0.1$ and $W/H=0.2$, which closely resembles the arterial prostheses. This geometry is similar to that in the mumerical study for the application to arterial prostheses presented by Savvides et al., [10] but they can not directly compared. The Womersley number α as a function of the Reynolds number Re and the period of oscillation T (cf. section 3) is equivalent to the Reynolds number when $T=$const., so only the case of the period of oscillation $T=10$ is studied for the range of Reynolds number $300 \leqq Re \leqq 2000$. The ratio of flow rate Q_1/Q_0 is also fixed at the same value as the study of Kawaguti et al.. [11]

Although the maiority of results of this study has already been reported in the Journal of JSME, [12] only a part of the results for the calculated range of Reynolds number was presented on account of restriction of the number of pages. The oscillating flow over the roughness elements is very interesting not only from the medical point of view, but also from a purely fluid mechanical point of view. In this paper, particular attention is paid to the oscillating flow pattern, and all that behavior of vortex in the roughness groove, on the effect of roughness and the dependence on the Reynolds number or the Womersley number. The influence of roughness on the vascular lesion is discussed from the structure of pulsatile flow over a rough wall.

2 . Basic Equation and Calculation Model

Consider the unsteady blood flow in a two−dimensional channel with roughness elements. The blood is assumed to be a Newtonian fluid and all the walls of the channel are considered to be rigid. The governing equations for the two−dimensional flow of a Newtonian fluid are the two momentum equations (Navier−Stokes equation) and the continuity equation. By introducing the vorticity ω (where $\omega=\partial u/\partial y-\partial v/\partial x$) and the stream function ψ (where $u=\partial\psi/\partial y$, $v=-\partial\psi/\partial x$) which automatically satisfies the continuity equation, and eliminating the pressure from the Navier−Stokes equation by the cross−differentiation, we obtain the vorticity transport equation in the well−known form

$$\frac{\partial\omega}{\partial t}=\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right) \tag{1}$$

$$\omega=-\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right) \tag{2}$$

where Re is the Reynolds number, $Re=UH/\nu$, U denotes the mean fluid velocity in the channel, H the width of the channel and ν the kinematic viscosity of the fluid, respectively. Equations (1) and (2) are expressed in the non−dimensional forms using the reference length and velocity being H and U, and non−dimensional time t is normalised by the advective time scale H/U.

The calculation model is the same as for the steady and pulsatile flow given in the previous paper,[8),9),12)] hence only the outline on the model is described as follow; the several roughness elements are mounted on the wall of two−dimensional channel in the region of plane−Poiseuille flow (Fig. 1), the roughness elements are of the repeated ribs type with the spacing $W=0.2H$ and the roughness height $K=0.1H$, that is, the

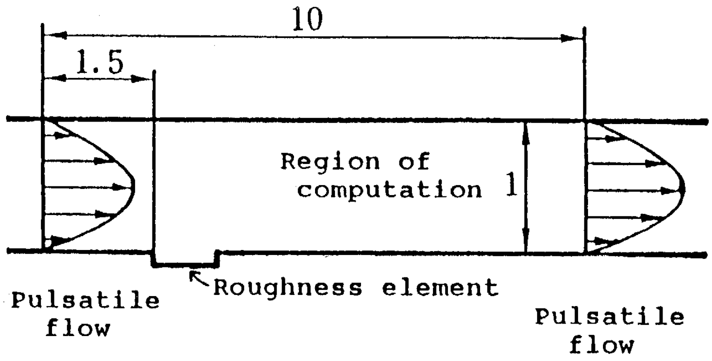

Fig. 1 Calculation model.

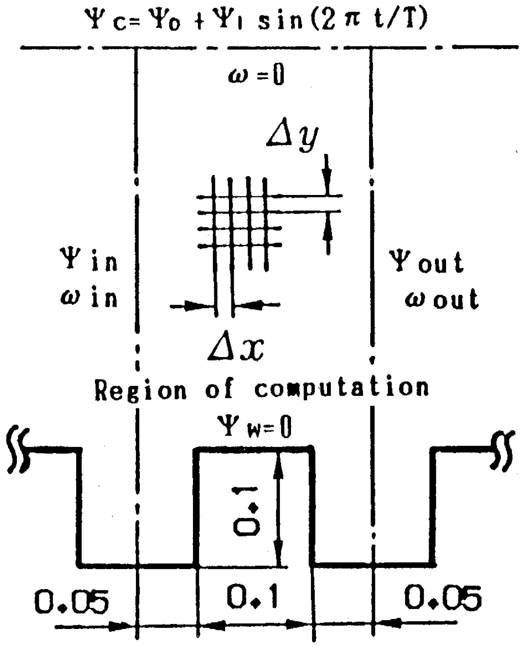

Fig. 2 Region of computation (in and out indicate the upstream and downstream, respectively)

pitch ratio is $W/K=2$ (Fig. 2) . From the numerical results of the steady flow over the roughness elements by authors,[8),9)] it is natural to assume that the flow is symmetric with respect to the center–line, and periodic corresponding with roughness periodicity. So we shall consider only the lower half of the finite region which the length along the channel is a roughness spacing $W=0.2H$ (Fig. 2). The position of connecting boundary (in –flow and out–flow boundaries in that computational region) can be determined arbitrarily. But, it seems that the selection of connecting boundaty in the neighborhood of convex sharp corner gives rise to undesirable effects for the numerical solution, so its position is chosen at the center of roughness groove (Fig. 2) .

The pulsatile flow is expressed by applying the two–dimensional Womersley's flow which is the flow through a two–dimensional channel when the oscillating pressure gradient along the channel is defined as

$$-\frac{\partial p}{\partial x}=A+B\cos(2\pi t/T) \quad (3)$$

where A and B are constants, and T is the period of oscillation. For the channel without the roughness elements, this problem can be easily solved analytically as described in appendix. For the case of the channel with roughness elements, the analytical solution on the connecting boundaty in the computational region can not be obtained, hence the values of ψ and ω on the connecting boundary are nor given explicitly. For that reason, the connecting boundary values are given by using the numerical method of periodic flow boundary condition.

In the computational region (Fig. 2), the boundary conditions are given as follows :

(i) $\psi_w=0$ at the wall by the non–slip condition,

(ii) $\psi_{in}=\psi_{out}$, $\omega_{in}=\omega_{out}$ on the in–flow and oue–flow boundarys by the periodic flow condition,

(iii) $\psi_c=\psi_0+\psi_1\sin(2\pi t/T)$, $\omega_c=0$ on the center–line by the conditions of symmetrical flow with respect to the center–line and pulsatile flow. Here, ω_c is equivalent to the half of flow rate through the channel in which ψ_0 and ψ_1 are the corresponding to the value of mean flow of averaging time T and amplitude of oscillating flow, respectively. The stream function on the center–line ψ_c as a function of time can be obtained by the integrating the velocity u (solution of Womersley's flow) over the channel width (cf. appendix).

The difference equations are derived from the differential eqs. (1) and (2) , using the forward–time and centered–space differences, as the FTCS method. The mesh constant Δh and the time step Δt are taken to be $\Delta h=0.01$ and $\Delta t=0.005$ throughout the numerical work in this paper. Using the FTCS difference equations of eqs. (1) and (2) , the numerical computation is carried out according to the following steps:

(i) At the time $t=0$, we put the values of ψ and ω for the steady flow, that is, these values of initial condition are used numerical results for the steady flow under considering the Reynolds number.

(ii) Proceed to the next time $t=\Delta t$, and the boundary value of ψ_c on the center–line is replaced with the calculated value of ψ_c at $t=\Delta t$ determined from the equation of ψ_c as a function of time (cf. boundary condition (iii)) .

(iii) At the time $t=\Delta t$, calculate the values ψ from the inflow boundary to the out–flow boundary in the computational region using the difference equation of eq. (2).

(iv) The values of ψ on the in–flow boundary are replaced with the newly calculated values of ψ on the out–flow boundary.

(v) Iterative procedure (steps (iii) and (iv)) for the values of ψ is continued until a sufficient convergence is accomplished; i. e. , until the differences in the values of ψ at the succesive loops at the several points (to be ten points in this work) become less than 1×10^{-5}, simultaneously.

(vi) Calculate the values of ω on the wall and then in the computational region at the time $t=\Delta t$, in turn using the difference equations of eqs. (2) and (1), respectively.

(vii) The above procedure (steps (ii) ~(vi) is continued over a period of oscillation T.

The computational loop mentioned above is repeated until a convergence criterion for a state of pulsatile flow can be satisfied. The convergence criterion was defined as $|\omega^{(n+1)}-\omega^{(n)}|\leqq 10^{-5}$ wher n is the period ondex such that $\omega^{(n)}=\omega(nT)$. This numerical procedure for the pulsatile flow is almost the same as the previous papers by authors. [8),9),13)]

3. Results and Discussions

The calculation of the pulsatile flow in the two−dimensional channel with roughness elements was performed for the cases of the oscillating period of $T=10$, the amplitude of oscillating flow rate of $Q_1=0.5Q_0$ and the Reynolds number of $Re=300$, 600, 1200 and 2000.

The Womersley number α is defined as

$$\alpha=0.5H\,(2\pi/\nu\tilde{T})^{1/2} \tag{4}$$

where $\tilde{T}$ is dimensional period of oscillation. By introducing the Reynlods unmber $Re=UH/\nu$ and non−dimensional period of oscillation $T=\tilde{T}/(U/H)$, we obtain the α as a function of Re and T,

$$\alpha=(0.5\pi Re/T)^{1/2} \tag{5}$$

Therefore, each Reynolds number of $Re=300$, 600, 1200 and 2000 for $T=10$ coresponds to the Womersley number of $\alpha=6.85$, 9.71, 13.73, and 17.72, respectively. These ranges of Re or α are closely related to the physical condition for the blood flow in the human main artery.

In Figs. 3∼5, the oscillating flow pattern is presented by drawing the instantaneous stream lines at the interval time of $T/8=1025$ in a flow cycle. In these figures, the scale in y−direction was doubled relative to the scale in x−direction and the stream line for the value of $\psi=0$ was indicated by the dotted line.

The oscillating flow pattern for the case of $Re=300$ ($\alpha=6.85$) is shown in Fig. 3, and we will firstly discuss the pulsatile flow for this case. As the flow rate increases for t=0∼t=2.5, the development of recirculating stream lines (separation vortex) in the roughness groove is seen, but the location of separation vortex center dose not move in its groove. As the flow rate decreases, the decay of this vortex is seen, and the recirculating region grows in size and bulges to more than a full of the roughness groove (see for $t=2.5\sim t=5$ in Fig. 3). At $t=6.25$, this separation vortex gets half out from the roughness groove. As the flow rate approches the minimum volume at $t=7.5$, this separation vortex falls into decay in the roughness groove. Then the newly vortex in the roughness groove is developing with increasing flow rate for $t=7.5\sim t=10$. From the flow pattern described above, we consider that the flow in this case (α is small) can be regarded as quasi−steady flow.

Turning now Fig. 4 which is shown for the case of $Re=600$ ($\alpha=9.71$), we can see the effects on increasing α. As can be seen in Figs. 3 and 4, the flow patterns are quite little different for $t=0\sim t=5$, but the difference of the flow patterns at $t=5$ in the two figures is seen, that is, the separation vortex for the case of $Re=600$ gets half out from the roughness groove at the early stage sbout $T/8$ compared with the case of $Re=300$. After $t=5$, as the flow rate approaches the minimum volume at $t=7.5$, the separation vortex moves out from the roughness groove and eventually disappeares at t between

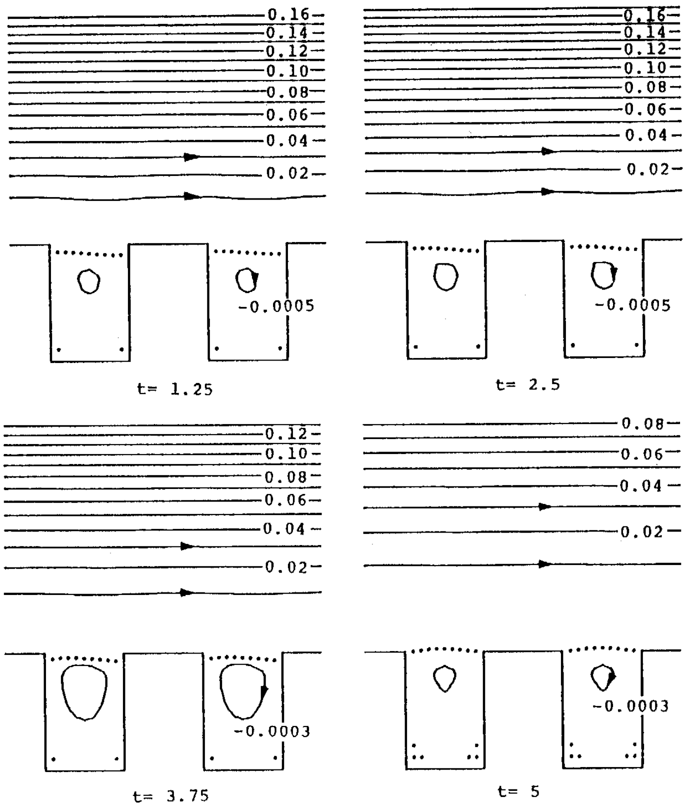

Fig. 3(a) Instantaneous stream lines of pulsatile flow ; Re=300, α=6.85,t=1.25~t=5.

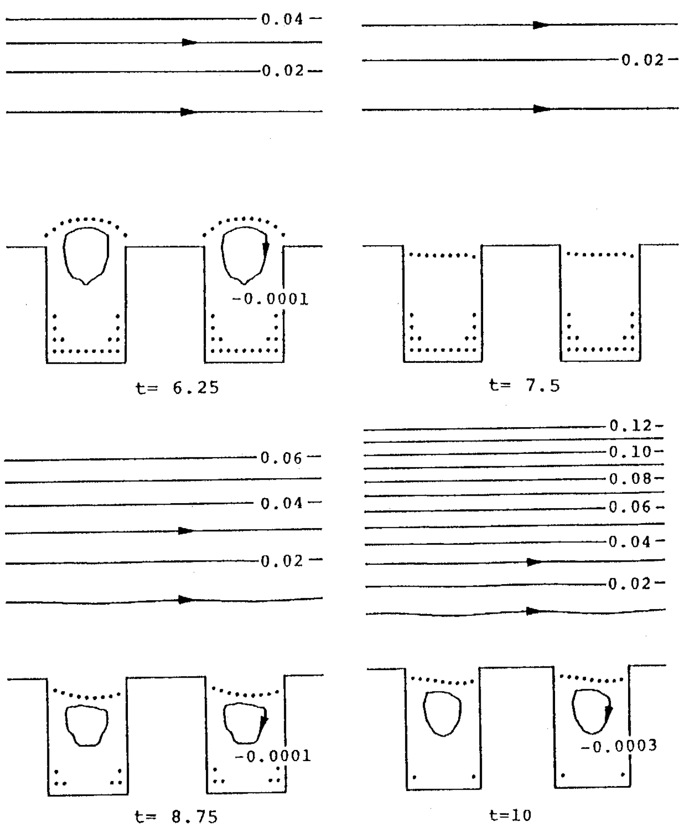

Fig. 3(b) Instantaneous stream lines of pulsatile flow ; Re=300, α=6.85,t=6.25~t=10.

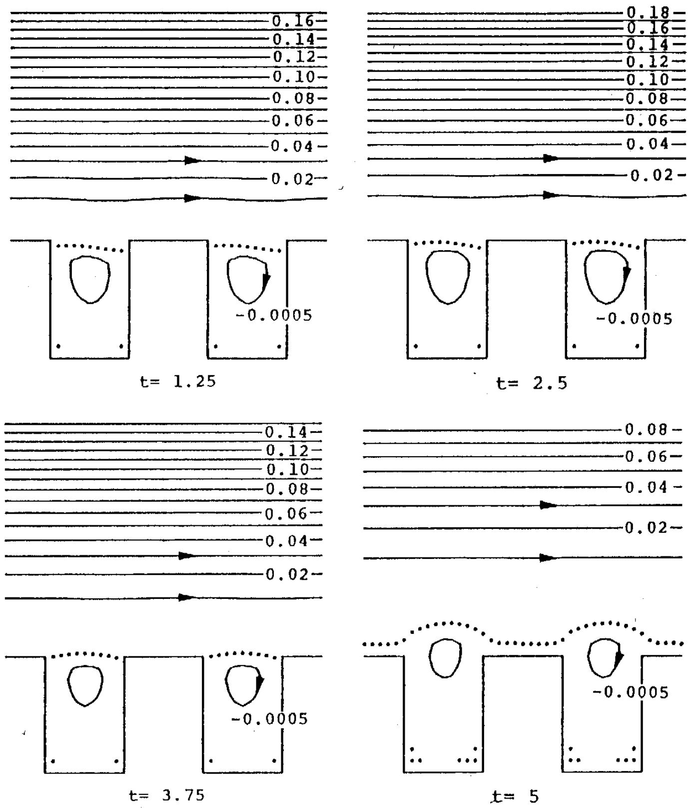

Fig. 4 (a) Instantaneous stream lines of pulsatile flow ; Re=600, $\alpha=9.71$,t=1.25~t=5.

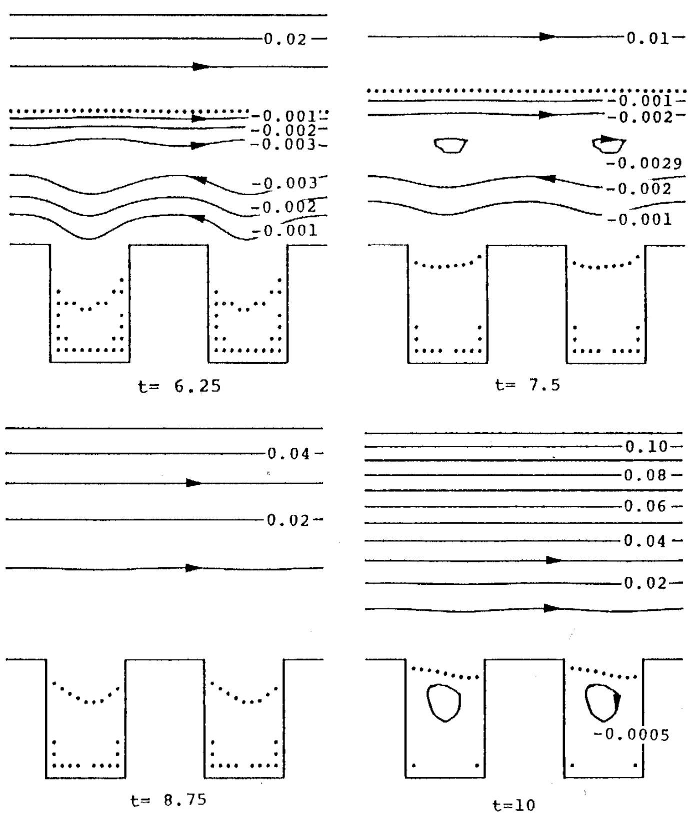

Fig. 4 (b) Instantaneous stream lines of pulsatile flow ; Re=600, $\alpha=9.71$,t=6.25~t=10.

7.5 and 8.75. During the times $t=5\sim t=7.5$, the back flow occures near the roughness wall and the flow in the roughness groove becomes stagnant (Fig.4) .

The flow patterns shown in Figs. 5 and 6 are for the cases of $Re=1200$ ($\alpha=13.73$) and $Re=2000$ ($\alpha=17.72$), respectively. It can be seen in Figs. 5 and 6 that the back flow near the roughness wall becomes cleary to be caused by the increasing the Womersley number. At $t=7.5$ in Fig. 5 and at $t=6.5$ and 7.5 in Fig.6, the reverse recirculating separation vortex which is formed by the back flow can be seen in the roughness groove. And this back flow washes out the fluid in the roughness groove. After the time $t=7.5$ when the flow rate becomes the minimum volume, the flow near the roughness wall starts to flow to the downstream, and the fluid in its groove is also washed out by the back flow at this stage. From the flow patterns described in Figs. 4～6,the flow for the cases of $Re=600\sim2000$ shows the oscillating flow patterns, snd the flow becomes more clear oscillating flow pattern as the Reynolds number Re (or the Womersley number α) increases.

These numerical results by authors agree qualitatively with the numerical results of Savvides.[10] From the flow patterns described above, it could be understood the effects of the Womersley number α on pulsatile flow, namely if α was small the flow would be quasi－steady flow, and observed the back flow near the roughness wall, the development and the decaying of separation vortex in the roughness groove and the existence of stagnant flow region and of flow stage to wash out in its groove for a pulsatile cycle. Further, we suppose that the stagnant flow exsits at the region of bottom in the roughness groove for the all stages of a pulsatile cycle.

4. Conclusions

From the numerical results and discussions given above, the following conclusions concerning the pulsatile flow through a two－dimensional channel with the roughness elements are summarized;

(1) The flow for the case of $Re=300$ ($\alpha=6.85$)is quasi－steady flow.

(2) The flow for the case of $Re=600$ ($\alpha=9.71$) shows the oscillating flow pattern, and the flow becomes more clear oscillating flow pattern as the Reynolds number Re (or the Womersley number α) increases.

(3) The back flow is observed near the roughness wall except for the case of $Re=300$ ($\alpha=6.85$).

(4) The development and the decaying of separation vortex in the roughness groove can be seen, and this separation vortex moves out from its groove and disappers during a pulsatile cycle except for the case of $Re=300$.

(5) The flow to wash out the fluid in roughness groove exists for a pulsatile cycle except for the case of $Re=300$.

(6) There is the stagnant flow region in the roughness groove, especially at the bottom in its groove.

It is well known that the atheroma and the throbsis are encounterd most frequently where the flow separation can occurs, as evidenced by the works of Fry(1968),[2] Caro et al. (1971)[3] and Fox et al. (1966).[4] The blood clotting (thrombus formation) is more likely in the region of stasis or the region of flow not washed out at each cycle of the

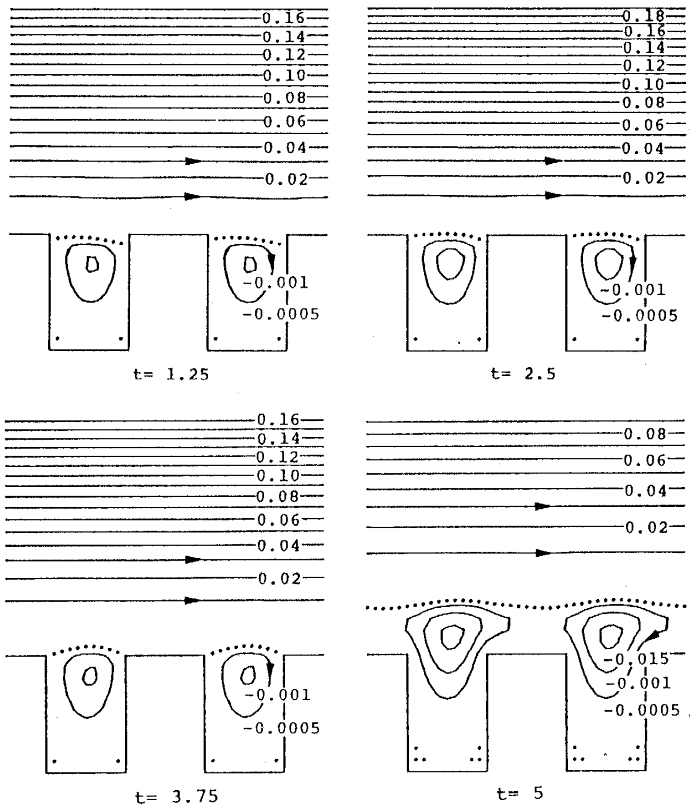

Fig. 5(a) Instantaneous stream lines of pulsatile flow ; Re=1200, $\alpha=13.73$,t=1.25~t=5.

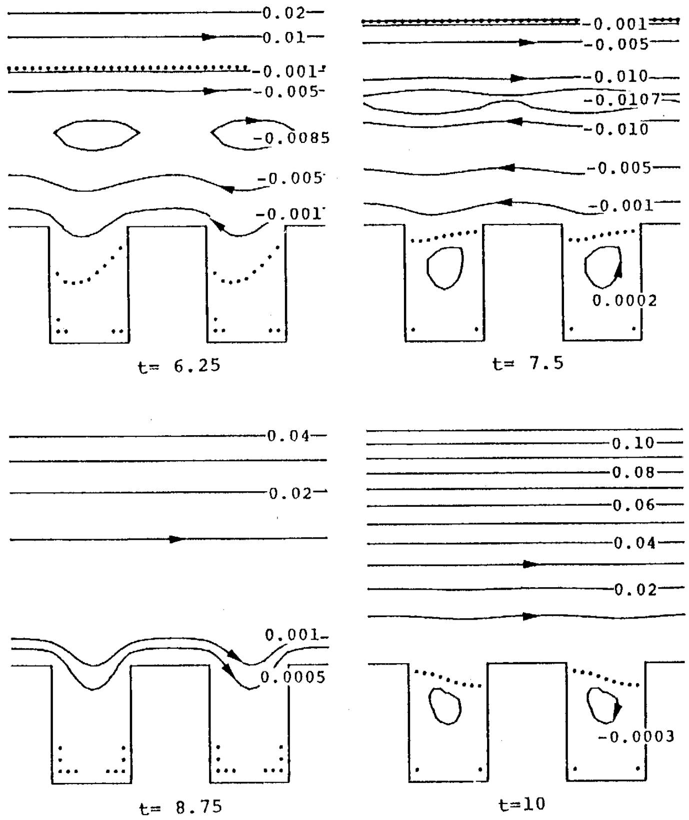

Fig. 5(b) Instantaneous stream lines of pulsatile flow ; Re=1200, $\alpha=13.73$,t=6.25~t=10.

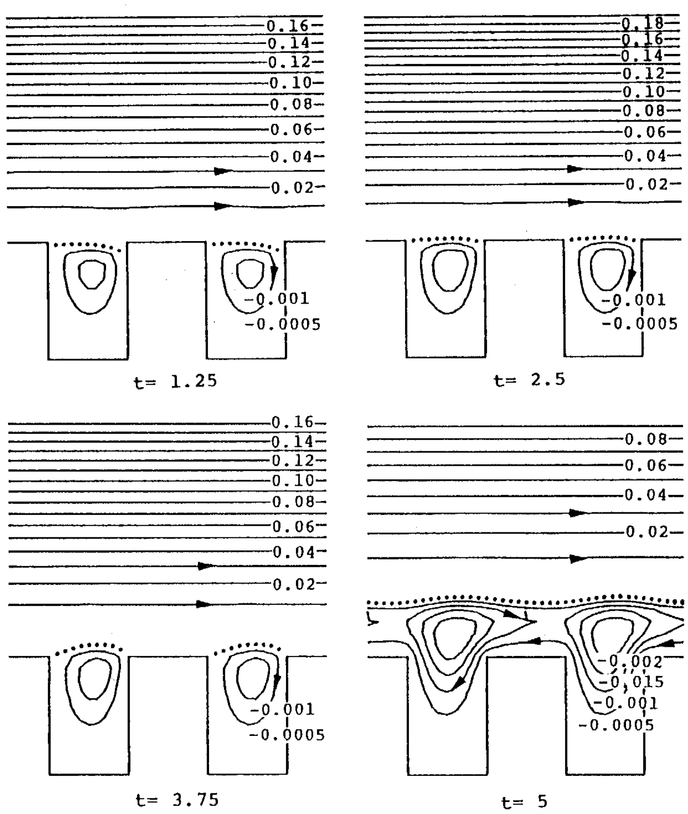

Fig. 6(a) Instantaneous stream lines of pulsatile flow ; Re=2000, $\alpha=17.72$, t=1.25~t=5.

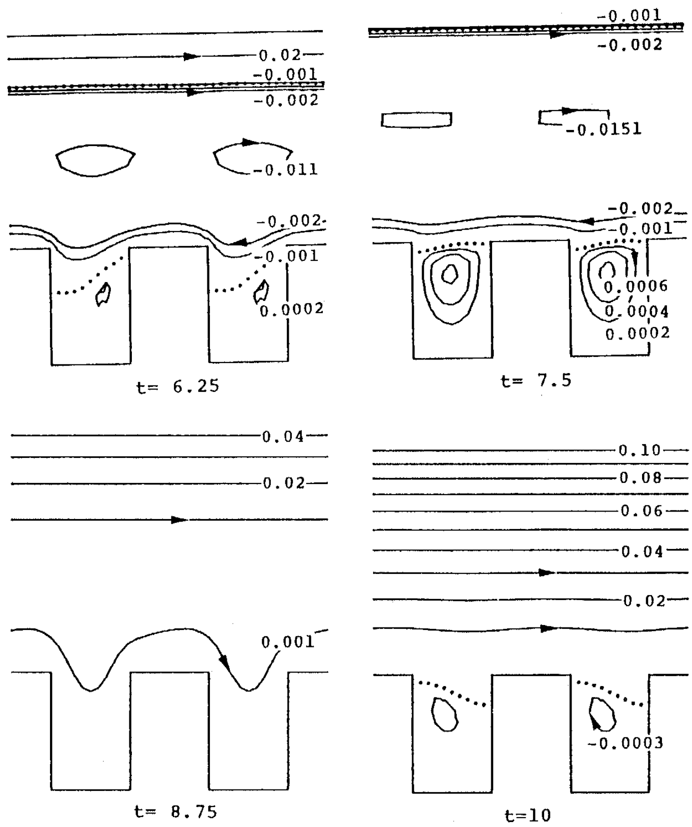

Fig. 6(b) Instantaneous stream lines of pulsatile flow ; Re=2000, $\alpha=17.72$, t=6.25~t=10.

heart beat. From this point of view, our numerical results suggest that the blood flow over the roughmess wall is likely to produce the crotting of blood because of a stagnant flow region in the roughness groove.

The authors would like to acknowledge the helpful suggestions and encouragements of Professor I. Nakamura of Nagoya University, Department of Mechanical Engineering.

References

1) F. B. Gessner : Circ. Res., **33** (1973), 259.
2) D. L. Fry : Circ. Res., **22** (1968), 165.
3) C. G. Caro, J. F. Fitzgerald and R. C. Schroter : Proc. R. Soc. Lond., **B177** (1971), 109.
4) J. A. Fox and A. E. Hugh : Brit. Heart J., **28** (1966), 388.
5) T. Azuma and T. Fukushima : Biorheology, **13** (1976), 337.
6) C. G. Caro : INSERM,**78** (1978), 33.
7) T. Fukushima : J. JSME, **88** (1985), 71.
8) S. Nakanishi and H. Osaka : Trans. JSME, Ser. B, **52** (1986), 3930.
9) S. Nakanishi and H. Osaka : Memoirs, Faculty Eng., Yamaguchi University, **39** (1988),
10) C. N. Savvides and J. H. Gerrard : J.Fluid Mech., **138** (1984), 129.
11) M. Kawaguti and A. Hamano : J. Phys. Soc. Jpn., **49** (1980), 817.
12) S. Nakanishi and H. Osaka : Trans. JSME, Ser. B, **52** (1986), 3935.
13) S. Nakanishi and M. Kawaguti : J. Phys. Soc. Jpn., **52** (1983), 848.

Appendix

Consider the two−dimensional Womersley's flow through a straight channel of which dimensionless width is 1, when the oscillating pressure gradient along the x−axis

$$-\frac{\partial P}{\partial x}=A+B\cos(2\pi t/T) \qquad \text{(A. 1)}$$

is applied. This problem can be easily solved analytically(11), and the velocity is given as

$$u(y,t)=u_p(y)+u_s(y)\sin(2\pi t/T)+u_c(y)\cos(2\pi t/T) \qquad \text{(A. 2)}$$

$$u_p(y)=(1-y'^2)ReA/8$$

$$u_s(y)=(ReB/8\beta^2)(1-2c_1\sinh\beta y'\cdot\sin\beta y'+2c_2\cosh\beta y'\cdot\cos\beta y'),$$

$$u_c(y)=(ReB/8\beta^2)(2c_1\cosh\beta y'\cdot\cos\beta y'+2c_2\sinh\beta y'\cdot\sin\beta y') \qquad \text{(A. 3)}$$

where

$$\beta=(2\pi Re/8T)^{1/2},\quad y'=2y-1,$$

$$C_1=\sinh\beta\cdot\sin\beta/(\cosh2\beta+\cos2\beta),$$

$$C_2=-\cosh\beta\cdot\cos\beta/(\cosh2\beta+\cos2\beta) \qquad \text{(A. 4)}$$

Integral of $u(y)$ over the channel width gives the flow rate Q as a function of time

$$Q=Q_0+Q_1\sin(2\pi t/T+\delta) \qquad \text{(A. 5)}$$

$$Q_0=ReA/12,\quad Q_1=(\xi_1^2+\xi_2^2)^{1/2},\quad \delta=\tan^{-1}\xi_1/\xi_2 \qquad \text{(A. 6)}$$

where

$$\xi_1=(ReB/4\beta^2)[1-\frac{1}{\beta}(\sinh\beta\cdot\cosh\beta+\sin\beta\cdot\cos\beta)/(\cosh2\beta+\cos2\beta)],$$

$$\xi_2=(ReB/4\beta^2)[\frac{1}{\beta}(\sinh\beta\cdot\cosh\beta-\sin\beta\cdot\cos\beta)/(\cosh2\beta+\cos2\beta)]. \qquad \text{(A. 7)}$$

The mean flow rate Q_0 is chosen to be $Q_0=1$, then the constant A is determined to be equal to 12/ Re, and the value of constant B can be determined from giving the amplitude of flow rate Q_1/Q_0. Here, the calculation method in this paper dose not need to use the explicit value of B, and the time of starting point can be determined arbitrarily, so its point is chosen at the time of $t=-\delta T/2\pi$.