On the Criterion for Scour from Flows at Downstream of an Apron

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Abstract

In order to prevent the local scour from flows at downstream of an outlet and such constructions, frequently one build up an apron to protect the erosive bottom of river. Although for that reason of the complicated flow condition neighbouring there, it is usually to design and planning of the length of an apron in conformity with the experimental results used the model and the practical measurement about the similar construction.

In previous report where the development of the boundary layer in the wall-jet flows has become a theme of research, the author suggested the velocity distribution law, in which the influence of the turbulence in jet-like main flows to the boundary layer has been taken into consideration, and explained excellently the boundary layer growth in wall jet due to several jets by the proposed resistant law.

In this paper, concentrating on the standpoint of engineers, the author shall try to explaine the criterion that the sedimentary bed at downstream of an apron is not absolutely scoured by the action of flows, by means of the easy correction about the criterion for movement of sand grain in open channel flows with the velocity distribution of the boundary layer in wall jet, furthermore, in order to alleviate the labour on the trial and error calculation regard to the critical tractive force and the boundary layer growth, presented the graphycal solution about these calculation.

Introduction

The critical shear velocity is decided by the equilibrium condition between the fluid drag force acting on sand particle and it's resistant force. Because of the flow at neighbouring of sand particle are turbulent, the criterion for movement of sand grain is provided by the movement of sand grain received the intense force due to turbulence.

According to Egiazaroff's results, it can be considered that a certain relation between the intenselly turbulent force and the fluid drag force due to the characteristic velocity axis. Although there are still problems to adopt this relation for the flow in which the velocity distribution law and the resistance law differ to that of the open channel flow.

A precise explanation for the turbulent characteristics within the boundary layer in wall jet has not been made so sufficiently as so evaluate the tractive force acting on sand particle analytically, because of the boundary layer flow in wall jet is fairly complicated flows depended on the turbulence in jet-like main flows.Furthermore, the ex-

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pression for the critical shear velocity may include the parameter as to the turbulence (eddy viscosity) in main flows, so that it is expected that the formula in regard to the criterion for scour at downstream of an apron are very complicated form. Consequently, concentrating on the standpoint of engineers, it is not meaningless works to examine the criterion for movement of sand particle in wall jet at downstream of an outlet, by means of to correct the critical shear velocity on open channel with the author's suggested velocity distribution in previous report.

Experimental apparatus and method

Experiments were carried out in an acrylic resin channel of 5.0 m length, 0.2 m width and 0.7 m height. The channel are provided with the jet issuing equipment with the slot of 1.07 cm thickness and 20 cm width at upstream end and the moving weir to adjust the water depth in channel at downstream end.

A smooth surface apron was made of an acrylic resin plate of 10 mm thickness and 20 ~ 80 cm length, and set up it on the floor which has adjustable supports. The roughness of an apron surface was simulated by using sand grain of $d=0.54$, 1.09, 2.18 and 3.09 mm in medium particle size. These sand grains were glued to an apron in a single layer.

A series of experiments have been carried out with suitable combinations of eight length of apron added four roughness, four size of sediment particle at downstream of an apron, and four abrupt drop (distance from the surface of an apron to the center line of the slot are 0.5, 2, 4 and 6 times of the slot thickness).

It is desirable to determine the criterion for the movement of sand grain from flows with the relation between a number of moved sand particle and the issuing velocity. The initial condition of the sand layer and the state of acceleration of issuing velocity have an effect on a number of moved sand grain, so that technically it is difficult to make account of a number of moved sand particles with accuracy and certainty. Therefore, in this experiments, the movement of sand grains in both state of relatively unstable and stable have been decided by the observation of plural experimenter, and the issuing velocity on the criterion of movement of sand grain has been defined with the avelaged value of issuing velocity in both state.

Experimental results and dicussion

The main parts of the experimental apparatus and important symbols used in this report are shown in Fig. 1.

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Fig. 2 shows experimental results with smooth and rough surface apron. The line in figure are calculated results used of Shields's critical shear velocity and the author's suggested resistance law, and the broken lines were shown the results proposed by Tsuchiya experimentally and theoretically. The agreement between the experi-

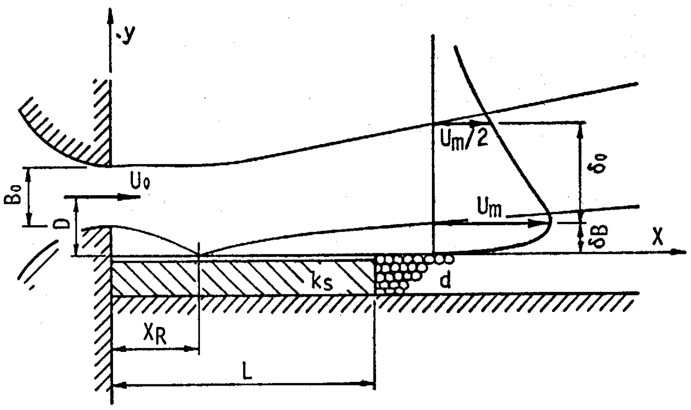

Fig. 1 Experimental apparatus and important symbols.

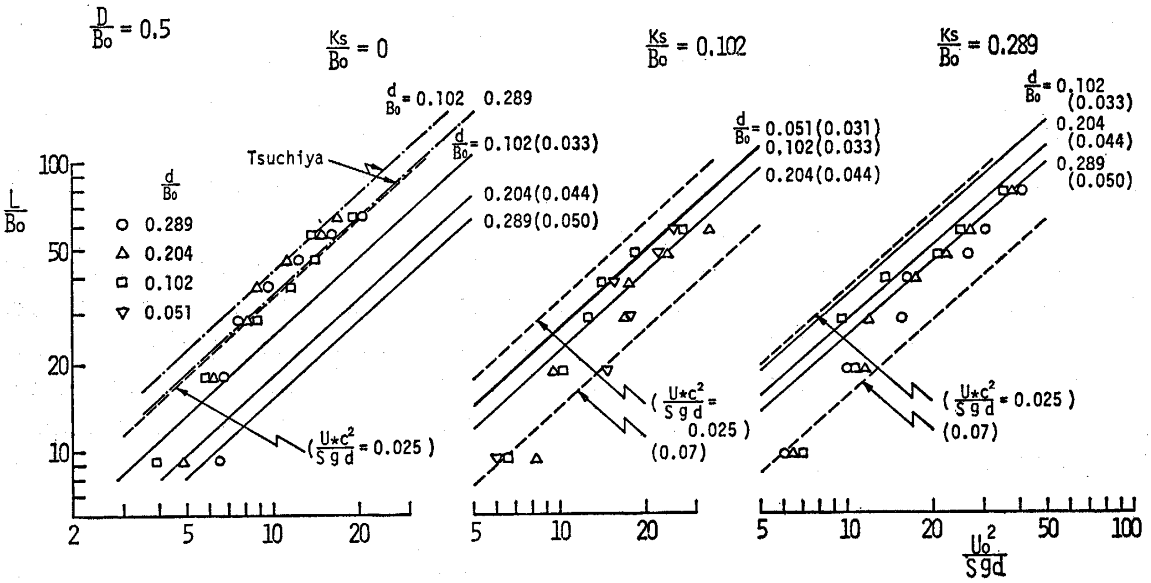

Fig. 2 The criterions for scour at downstream of an apron.

mental results and Tsuchiya's results is fairly good. Consequently, our subjectively decision agree approximately with Tsuchiya's decision.

It is usefull for discuss to estimate practical value of the critical shear velocity, so that, the calculated results in the case of $U_*c^2/sgd=0.025$ and 0.070 are shown by dotted lines, and the figures in parenthesis are the value of U_*c^2/sgd used in calculation.

In open channel flows, the critical shear velocity is given as follows.

$$\frac{U_*c^2}{sgd}=K\frac{\mu}{cf}\left(\frac{U_*c}{Ud}\right)^2=F\left(\frac{U_*c\cdot d}{\nu}\right) \qquad (1)$$

Where, U_*c: critical shear velocity,
s : specific gravity of sand grain in water,
g : acceleration of gravity,
d : medium particle diameter of sand grain,
μ : friction coefficient of sand grain,

c : resistance coefficient of sand grain,
f : effect of turbulent force on the body force,
Ud : typical velocity acting on sand grain,
K : experimental constant.

The function F in terms of Reynold's number has been decided by Shields's, Kurihara, Iwagaki and other investigators experimentally and theoretically.

As usually, the mean velocity at height in proportional to medium diameter of sand grain from wall surface are applied to the typical velocity, by which the drag force acting on sand particle are evaluated.

If the velocity distribution at vicinity of sand grain is differ, the value of Ud/U_* (Ud is the typical velocity) differ each other. But, making a clear the distribution at vicinity of sand grain, it is possible to correct the critical shear velocity by the same method as Egiazaroff's. Although, as mentioned previous, there are still problems to adopt the Egiazaroff's correct for boundary layer flows in wall jet.

Because of the criterion for movement of sand grain are decide by the movement of a particle which receive the intense turbulence. The value of f is prescribed by the intensity of turbulence for the typical velocity and the velocity gradient at vicinity of a particle, but the important parameter of f is the former.

In case of that a sand grain diameter is almost the same degree as roughness of an apron which provide the velocity distribution, flows at vicinity of a sand particle are the turbulent sub-layer, in where the turbulence are originated in roughness. So that the mean velocity distribution in the outer-layer does not take part in the relation between the turbulence at neighbouring of sand particles and Reynolds's number $U_* \cdot d/\nu$.

Rewrite Eq. (1) as follows.

$$K\frac{\mu}{cf}=F\left(\frac{U_{*}c\cdot d}{\nu}\right)\cdot\left(\frac{Ud}{U_{*}c}\right)^{2} \tag{2}$$

And let us consider about the value of Eq. (2).

Because of Eq. (2) is the relation for open channel flows, put $E=1$ in the author's suggested velocity distribution law in previous report (Eq. (30)), and support that Reynolds's number $R_*=\kappa\cdot U_*\cdot\delta_B/\nu$ is great relatively, then the typical velocity acting on sand particle ($y=d$) are given as follows.

$$\frac{Ud}{U_*}=\frac{U_*\cdot y_1}{\nu}+\frac{1}{\kappa(1-d/\delta_B)}=Ar\left(\frac{U_*\cdot d}{\nu}\right) \tag{3}$$

Where, y_1 : viscous sub-layer thickness,
δ_B : boundary layer thickness,
Ar: roughness function in terms of $U_*\cdot ks/\nu$ for Nikuradse's sand roughness.

$$Ar\left(\frac{U_*\cdot ks}{\nu}\right)=8.50+0.744\left(\frac{U_*\cdot ks}{\nu}-3.3\right)^{\frac{1}{2}}\exp\left\{-0.077\left(\frac{U_*\cdot ks}{\nu}-3.3\right)\right\} \tag{4}$$

The function F increase as $U_* \cdot d/\nu$ increase from 20 to 100, and is constant in $U_* \cdot d/\nu > 100$. On the other hand, the function Ar decrease as roughness Reynolds's number increase from 10 to 100, and is constant in the range of the complately rough regime. Accordingly, in the range of that the roughness Reynolds's number are greater than 20, the value of Eq. (2) which is given by the product of a increasing function by a decreasing function as the roughness Reynolds's number increase are almost constant.

In order to avoid the confusion, let us take the triple suffix. The first suffix denote the influence of turbulence in main flows (1 for open channel flows), the second denote the roughness of an apron, the last suffix denote diameters of sand grain which consist the movable bottom at downstream of an apron.

The fact as are stated above is expressed as follows.

$$\left(K\frac{\mu}{cf}\right)_{1dd}=\left(K\frac{\mu}{cf}\right)_{1kk} \tag{5}$$

On the other hand, from expreimental results in Fig. 2.

$$\left(K\frac{\mu}{cf}\right)_{Edd}\fallingdotseq\left(K\frac{\mu}{cf}\right)_{1dd} \tag{6}$$

$$\left(\frac{U_*c^2}{sgd}\right)_{Edd}\fallingdotseq\left(K\frac{\mu}{cf}\right)_{Edd}\left(\frac{U_*c}{Ud}\right)^2_{Edd}\fallingdotseq F\left(\frac{U_*c\cdot d}{\nu}\right) \tag{7}$$

In case of that the sand grain diameter is almost the same as roughness of an apron, the flows at vicinity of a sand particle are flows in the turbulent sub-layer, so that, the value of $\{K\cdot\mu/(c\cdot f)\}$ which is given as an averaged (integrated) value around a particle can be writen as follows.

$$\left(K\frac{\mu}{cf}\right)_{Ekd}\fallingdotseq\left(K\frac{\mu}{cf}\right)_{1dd}\fallingdotseq\left(K\frac{\mu}{cf}\right)_{Edd} \tag{8}$$

With this approximation, the criterion for movement of sand grain at downstream of an apron is given as follows.

$$\begin{aligned}\left(\frac{U_*c^2}{sgd}\right)_{Ekd}&=\left(K\frac{\mu}{cf}\right)_{Ekd}\left(\frac{U_*c}{Ud}\right)^2_{Ekd}\\&=F\left(\frac{U_*c\cdot d}{\nu}\right)\left(\frac{Ud}{U_*c}\right)^2_{Edd}\Big/\left(\frac{Ud}{U_*c}\right)^2_{1kd}\end{aligned} \tag{9}$$

The correct terms in above equation are given by Eq. (30) in previous report (simplified solution) as follows.

$$\left(\frac{Ud}{U_*c}\right)_{1dd}=Ar\left(\frac{U_*c\cdot d}{\nu}\right) \tag{10}$$

$$\left(\frac{Ud}{U_{*}c}\right)_{Ekd}=Ar\left(\frac{U_{*}c\cdot d}{\nu}\right)-\frac{1}{\kappa}\left(\frac{1}{1-ks/\delta_B}-\frac{d/ks}{1-E\cdot ks/\delta_B}\right) \quad \text{for } \frac{d}{ks}\leq 1$$

$$\begin{aligned}&=Ar\left(\frac{U_{*}c\cdot d}{\nu}\right)-\frac{1}{\kappa}\frac{(1-E)ks/\delta_B}{(1-ks/\delta_B)(1-E\cdot ks/\delta_B)}\\&+\frac{1}{\kappa}\left\{\ln\frac{d}{ks}+\frac{1-E}{E}\ln\left(\frac{1-E\cdot d/\delta_B}{1-E\cdot ks/\delta_B}\right)\right\}\end{aligned} \quad \text{for } \frac{d}{ks}\geq 1 \tag{11}$$

In case of a smooth apron, let us write $ks=0$ formally, then

$$\left(\frac{Ud}{U_{*}c}\right)_{EOd}=As+\frac{1}{\kappa}\left\{\ln\frac{U_{*}c\cdot d}{\nu}+\frac{1-E}{E}\ln\left(1-E\frac{d}{ks}\right)\right\} \tag{12}$$

If we express the function F in terms of $U_{*}\cdot d/\nu$ with the curve of the critical shear velocity which has been given by Shields's, we can estimate the length of an apron in state of the criterion for scour from flows at downstream of an apron.

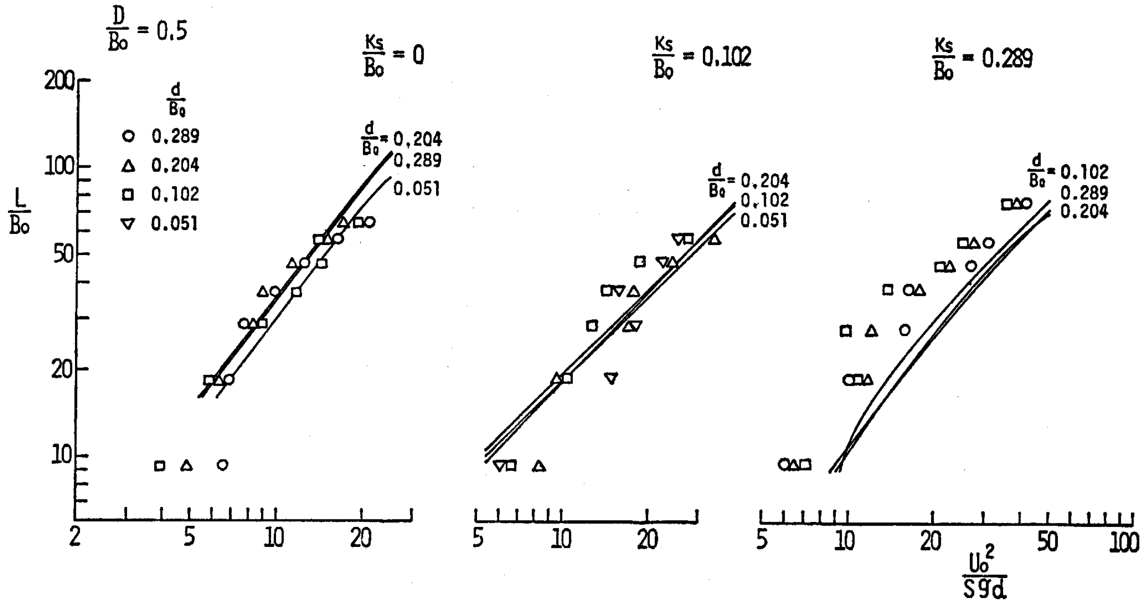

Fig. 3 Comparison between the theory and experimental results.

Fig. 3 indicate the calculated results with experimental values.

In case of a smooth apron, in spite of the conventional exchange on the correct terms, the calculated results agree well with experimental results beyond expection. In case of rough surface apron, experimental value are very scattered, but the tendency of calculated results agree well with that of experimental results.

It becomes clear from Fig. 3 that, if we add the roughness of the same order of magnitude as sand grain size of sedimentary bed at downstream of apron, the critical length of an apron in state of that the river bottom at dowmstream are not absolutely scoured by action flows are almost half as long as that of the smooth apron.

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The experimental results used an abrupt droped apron is shown in Fig. 4. In Fig. 4, the curves are calculated results used the graphycal solution that is descrived later.

Except for the case of $D/B_0 = 1.89$ and $ks/B_0 = 0.289$, experimental results agree well with the calculated value. Futhermore, in case of the $ks/B_0 = 0.204$ which are omited from Fig. 4, an agreement between the experimental results and calculations are good as the same degree as in Fig. 4(a).

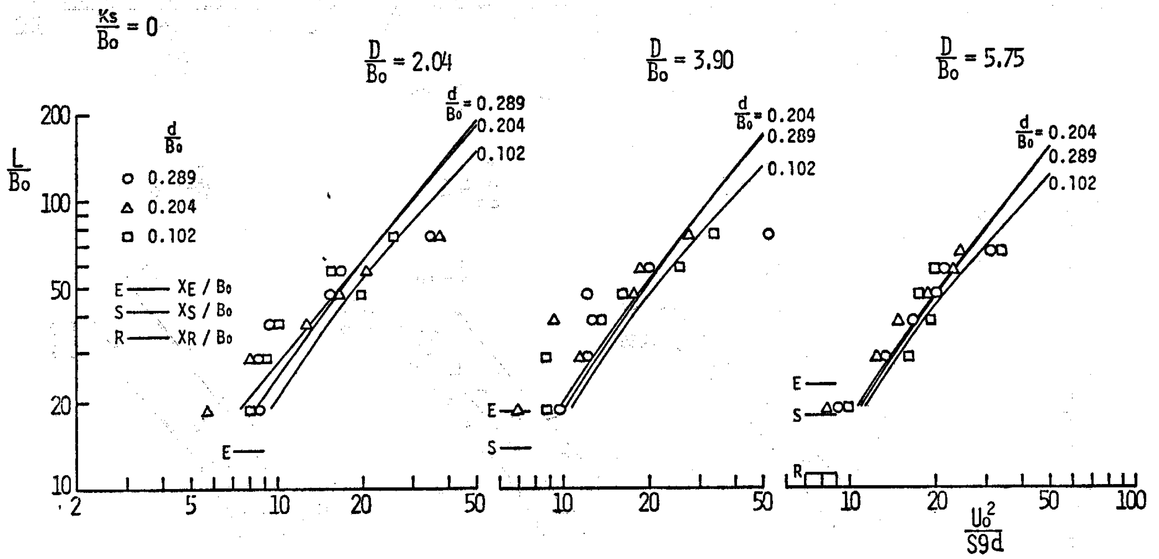
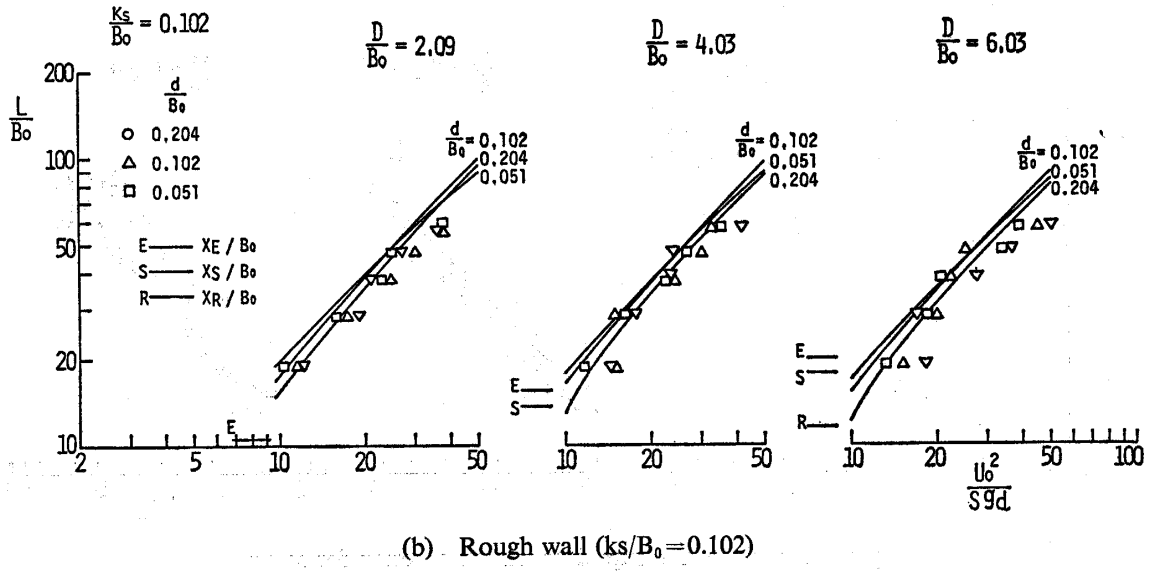
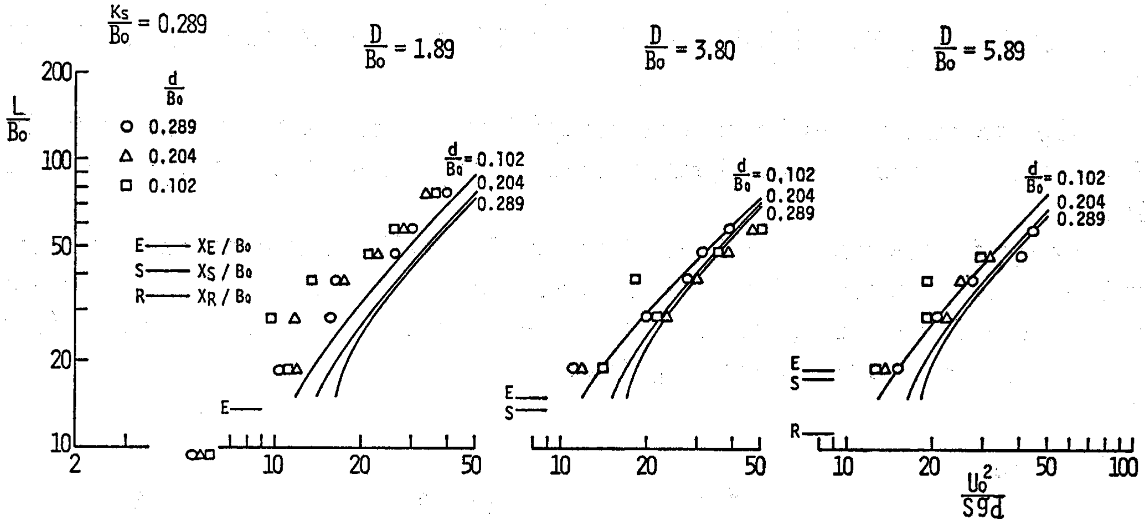


Fig. 4 The criterion for scour at downstream of an abrupt droped apron.
(a) Smoothwall



(b) Rough wall ($ks/B_0 = 0.102$)



(c) Rough wall (ks/B$_0$=0.289)

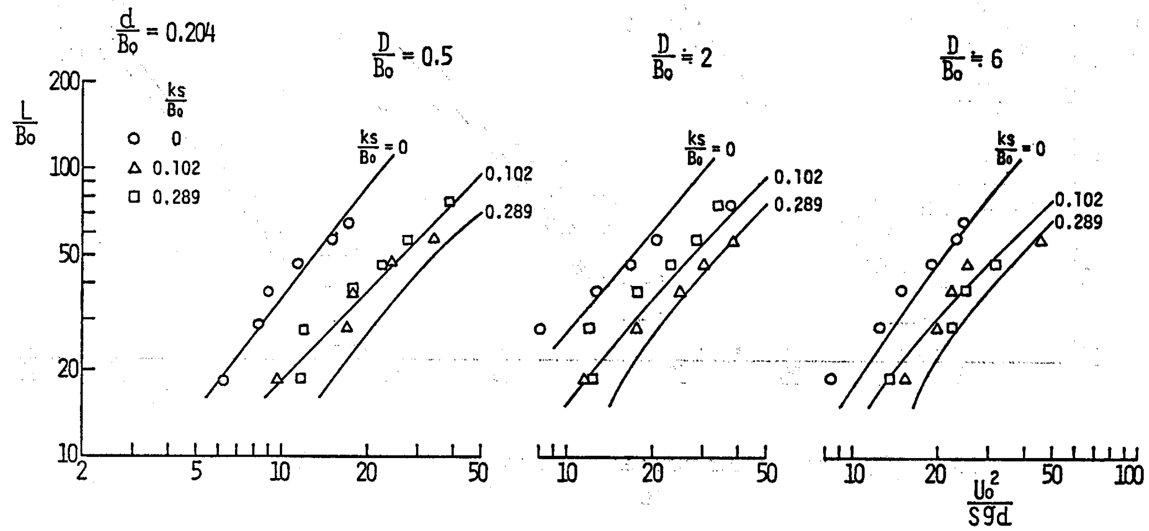


Fig. 5 The effect of apron roughness on the criterion for scour.

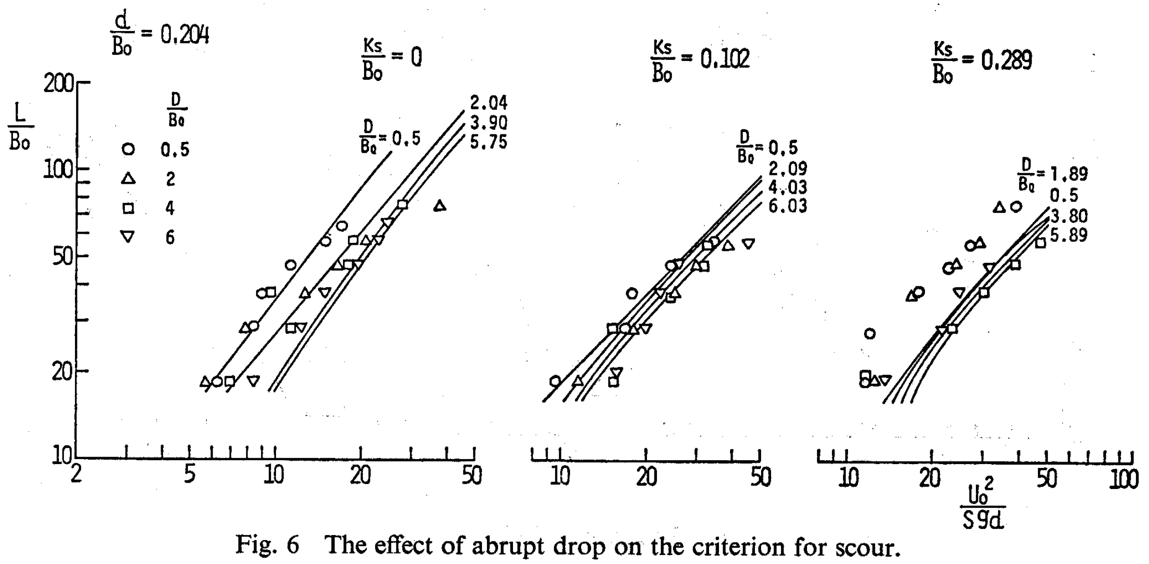


Fig. 6 The effect of abrupt drop on the criterion for scour.

Fig. 5 indicate the difference of the critical length of an apron for scour to the varied roughness of an apron. The critical length of an apron in both cases of $D/B=0.5$ and 6 are neary equal. Accordingly, it can be considered as there are little effect to short the length of an apron by using the abrupt drop.

In order to recognize that, the difference on the critical length of an apron depend on the abrupt drops are shown in Fig. 6.

THE GRAPHICALLY SOLUTION ABOUT THE CRITERION FOR SCOUR

In order to decide the criterion for scour at downstream of an apron, it is necessary to estimate the boundary layer development along an apron. A considerable amount of time and effort must be spend for this calculation involved trial and error. In order to alleviate the labour on this calculation, the theoretical and experimental results for wall jets in previous investigation are replaced by the conveniently approximations for practical aplication, and the author suggest the graphically solution about the critical length of an apron for scour at downstream there.

U_*/sgd and U_*/U_0 are rewriten as follows.

$$\frac{U_*^2}{sgd}=\left(\frac{U_*\cdot d}{\nu}\right)^2\Big/\frac{sgd^3}{\nu^2} \tag{13}$$

$$\frac{U_*}{U_0}=\left(\frac{U_*\cdot d}{\nu}\right)\Big/\left(\frac{U_0\cdot d}{\nu}\right) \tag{14}$$

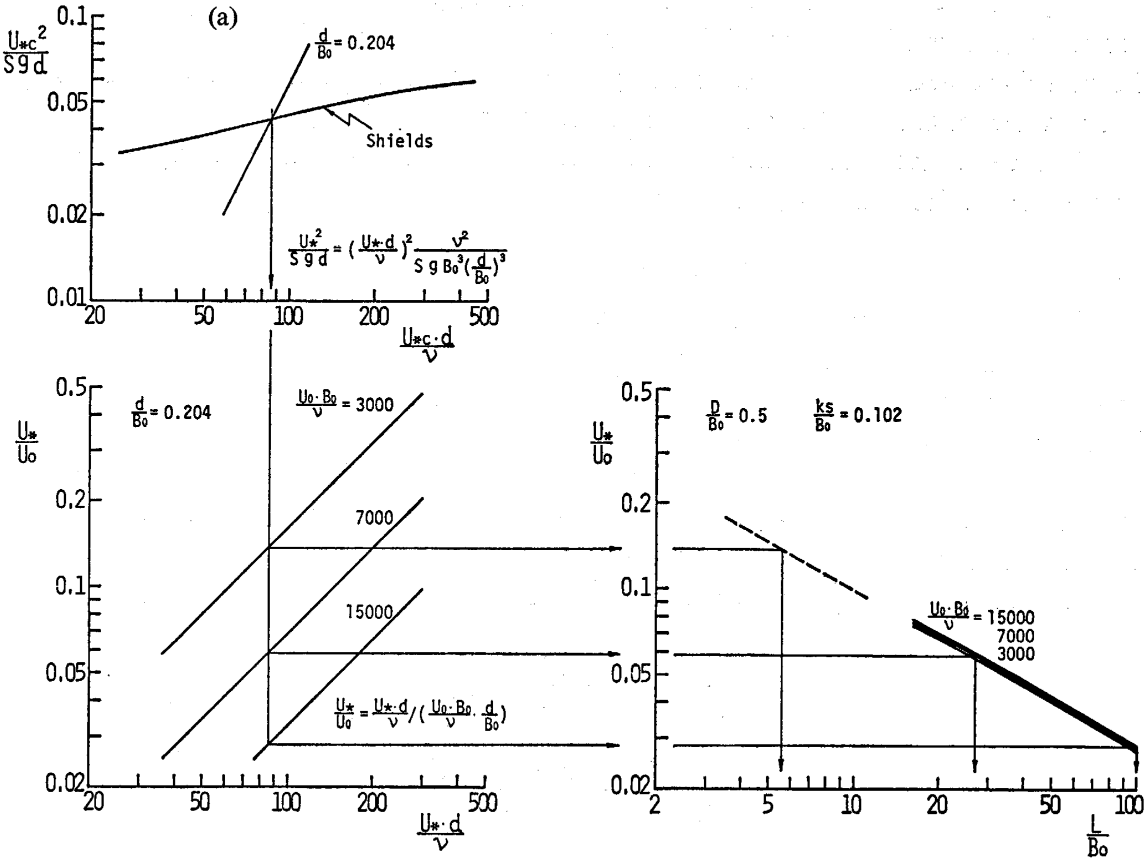
As shown in Fig. 7 (a) and (b), the relation between U_*^2/sgd or U_*/U_0 and $U_*.\cdot d/\nu$ can be expressed straight line in logarithmic graph paper.

If we decide the distribution of shear velocity (U_*/U_0) along an apron as showed in Fig. 7(c) and the relation between U_{*c}^2/sgd and $U_{*c}\cdot d/\nu$ as shown in Fig. 7(a), we can estimate the critical length of an apron by the method as are described below.

First of all, under assuming the length of an apron (L/B_0), one estimate the correct terms in Eq. (8) (Eq. (9) and Eq. (10)). Then, move parallel the straight line which express the relation U_*^2/sgd and $U_*\cdot d/\nu$ in Fig. 7(a) in the same direction as vertical axis at the distance of reciprocal magnitude as the correct terms.

As shown in Fig. 7, if we draw straight line parallel to axis in direction of arrow from the intersection of the parallel line and the curve which give the critical shear velocity suggested by Shields's, the intersection of straight line with arrow parallel to axis and the curve which express the relation between U_*/U_0 and L/B_0 in Fig. 7(c) is the critical length of an apron for scour. On repeating this procedure so that the estimated length coincide with the assumed length, the critical length of an apron was decided. On originary occasion, a convergency of this procedure is very good, and the number of times are two or three.

On determination of the relation between U_*/U_0 and L/B_0 in Fig. 7(c) and the correct term in Eq. (8), it is necessary to estimate the development of boundary layer. But there are the trial and error calculation in this estimation. So that, let us rewrite the results on wall jet in previous report by the convenient approximations.



(b) (c)

Fig. 7 Illustration for graphical solution about the criterion for scour.

[Wall jet]

Nondimensional eddy viscosity in main flow; This values are depended on δ_B/δ_0, and δ_B/δ_0 are depended on Reynolds's number $U_0 \cdot d/\nu$. The change of δ_B/δ_0 with Reynolds's number are so little as neglect, so that, nondimensional eddy viscosity in main flow are decided almostly with ks/B_0. Accordingly, we express this values by the function in term of ks/B_0. The ratio of the eddy viscosity in main flows (ε) to $Um \cdot \delta_0$ is related to ks/B_0 by following equation.

$$\alpha = \frac{\varepsilon}{Um \cdot \delta_0} = 0.0262\left\{1.0 - 1.88\left(\frac{ks}{B_0}\right)^{0.148}\right\} \tag{15}$$

Diffusion of main flow; For convenience sake, we summarize the results related with the diffusion of jet-like main flow from within previous investigation.

$$\frac{Um}{U_0} = \sqrt{\frac{x_E + x_0}{x + x_0}}, \qquad \frac{x_0}{B_0} = 4.32 \tag{16}$$

$$\frac{x_E}{B_0} \Big/ \left\{1 - 0.370 \frac{x_E}{B_0}\left(\frac{\nu}{U_0 \cdot x_E}\right)^{\frac{1}{5}}\right\} = \frac{x_*}{B_0} \qquad \text{for smooth} \tag{17}$$

$$\frac{x_E}{B_0}\Big/\left\{1-0.248\frac{x_E}{B_0}\left(\frac{ks}{B_0}\right)^{\frac{1}{4}}\right\}=\frac{x_*}{B_0} \qquad \text{for rough}$$

$$\frac{\delta_0}{B_0}=0.068\left(\frac{x}{B_0}+5.0\right) \tag{18}$$

Boundary layer development; Based on a particular aspect of the subject, with the exception of the zone of flow establishment, make an approximation with linear growth of boundary layer in the zone of established flow (see Fig. 16 in previous report), then the results is as follows.

$$\frac{\delta_B}{B_0}=\left\{0.01+0.027\left(\frac{ks}{B_0}\right)^{\frac{2}{3}}\right\}\left(\frac{x}{B_0}+22.0\right) \tag{19}$$

Coefficient of local skin friction; The calculated results used the simplified velocity distribution low which is suggested by the author is approximated by straight line on logarithmic graph paper safely. These straight lines can be expressed as follows.

$$Cx=\left\{0.0128+0.103\left(\frac{ks}{B_0}\right)^{\frac{1}{2}}\right\}\left(\frac{x}{B_0}\right)^{-\left\{0.237+0.15\exp\left(-100\frac{ks}{B_0}\right)\right\}} \tag{20}$$

[Wall jet due to a curve jet]

Nondimensional eddy viscosity in main flow; As mentioned above, if we express the values of α by the function in term of ks/B_0, the convenient approximation is as follows.

$$\alpha=0.0265\left\{1.0+1.49\left(\frac{ks}{B_0}\right)^{0.076}\right\} \tag{21}$$

Diffusion of main flow; In the same manner as well jet, we summarize the experimental and theoretical results with regard to the diffusion of jet-like main flowin the zone of established flow.

$$\frac{Uj}{U_0}=\sqrt{3.5\Big/\left(\frac{D}{B_0}+3.0\right)} \tag{22}$$

$$\frac{Um}{U_0}=Ku_1\cdot Ku_2\sqrt{15.4\Big/\left(\frac{x}{B_0}+5.9\right)} \tag{23}$$

Here, $$Ku_1=\left\{2.4\Big/\left(\frac{D}{B_0}+1.9\right)\right\}^{\frac{1}{8}}$$

$$Ku_2=A^{\left(\frac{d}{B_0}\right)^2/[4\{\left(\frac{d}{B_0}\right)^2+0.005\}]}$$

$$A=0.176\log\left(\frac{D}{B_0}\right)+0.553$$

$$\frac{\delta_0}{B_0}=0.068\left(\frac{x}{B_0}+5K_{D1}+K_{D2}\right) \tag{24}$$

Here,

$$K_{D1}=1.5^{(\frac{D}{B_0}-0.5)/\{(\frac{D}{B_0}-0.5)+0.001\}}$$

$$K_{D2}=7.5[2^{(\frac{d}{B_0})^2/\{(\frac{d}{B_0})^2+0.001\}}-1]$$

$$[2^{(\frac{D}{B_0}-0.5)^2/\{(\frac{D}{B_0}-0.5)^2+0.001\}}-1]$$

Boundary layer development; As are shown in Fig. 20 in previous report, in the zone of declaration, the boundary layer growth are depended almostly by ks/B_0 and are not affected very much by D/B_0 and $U_0 \cdot B_0/\nu$.

$$\frac{\delta_B}{B_0}=\left\{0.01+0.016\left(\frac{ks}{B_0}\right)^{\frac{2}{3}}\right\}\left\{\frac{x}{B_0}+10.0+27.0\left(\frac{ks}{B_0}\right)^{\frac{1}{3}}\right\} \tag{25}$$

The value of parameter E which express the effect of turbulence in main flow to boundary layer flow.

$$E=1-\frac{\alpha \cdot Um \cdot \delta_0}{\kappa \cdot U_* \cdot \delta_B}=1-\frac{\alpha \cdot \delta_0}{\kappa \cdot \delta_B}\sqrt{\frac{2}{Cx}} \tag{26}$$

are $-1 \sim -5$ usually.

It is possible to rewrite U_*/U_0 as follows.

$$\frac{U_*}{U_0}=\frac{U_*}{Um}\cdot\frac{Um}{U_0}=\sqrt{\frac{Cx}{2}}\cdot\frac{Um}{U_0} \tag{27}$$

Hence, using Eq. (15) ~ Eq. (20) or Eq. (21) ~ Eq. (27), it is possible to draw the relation between U_*/U_0 and L/B_0 in Fig. 7(c). Accordingly, the length of an apron in state of the criterion for scour are decided by above mentioned procedure.

Conclusions

It is desirable to make clear the criterion for movement of a sand grain at downstream of an apron in conformity with the considerations regard to the fluid force acting on sand particles. But the characteristics of turbulence within the boundary layer in wall jet has not been made so clear as to evaluate the tractive force acting on sand particle. Furthermore, the criterion for movement of sand grain has been decided by the subjectivity of the experiments.

In view of the facts as mentioned above, concentrating on the standpoint of engineers, the author suggested the easy correction for the critical shear velocity by the author's suggested velocity distribution low taken in the consideration about the turbulence in main flow in the same way as Egiazaroff's means, and showed the graphical solution regard to deciding the length of an apron in the state of the criterion for scour from flows, furthermore, to avoid the complicated calculation regard to the development of boundary layer, rewrote the results regard to the boundary layer growth and the diffusion of jet-like main flow in previous investigation by conventional approximations.

The experimental values are very scattered, although, experimental results can be explained well by the suggested convenient approximation. From these results, the correct for the critical shear velocity and the approximation regard to the boundary layer growth and the diffusion of main flow is accurate enoughly for practical usage.

In order to short the length of an apron under the condition that the sedimentary bed at downstream, it is more effective to install the roughness as the same order of magnitude as sediment particle than to increase the abrupt drop. By installing the roughness, the critical length of an apron are almost half to that of the smooth apron.

It is necessary the further investigation in conformity with the detailed experiments for the turbulent characteristics within the boundary layer, because of this investigation are the convenient approximation based on the standpoint of engineers.

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