$$\frac{\cdot (\overline{W} R^2/a \cdot n\pi/L)^3 x^6 + \cdots\cdots\cdots\cdots\cdots\}}{\cdot (\overline{W} R^2/a \cdot n\pi/L)^3 + \cdots\cdots\cdots\cdots\cdots\}} .$$

二重管内層流温度分布の一解析

村　川　勝　彌

$$Wz(r)\frac{\partial T}{\partial z} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right) \quad (1)$$

$(T)_{z=0}=0$　(2),　$(T)_{r=r1} = f_1(\theta,z)$　(3),　$(T)_{r=r2} = f_2(\theta,z)$　(4)

$$Wz(r) = 2W\frac{\{(r/r_1)^2 - 1\} - \{(r_2/r_1)^2 - 1\}(\ln r/r_1)/(\ln r_2/r_1)}{\{(r_2/r_1)^2 - 1\}/(\ln r_2/r_1) - \{(r_2/r_1)^2+1\}}$$

$$= A_0' + A'_1 r + A_2' r^2 + A_3' r^3 + A'_4 r^4 + \cdots\cdots \text{(Taylor展開)},$$

$Wz(x) = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + A_4 x^4 + \cdots\cdots\cdots$

$T = (t-t_0)/t_0$　(無次元) $= R(r)\cdot\phi(\theta)\cdot e^{in\pi z/L}$

$(k=in\pi/L)$,　$x = r/(r_2+r_1)$,　$R = x^m \cdot u$

$\therefore\ \phi_m(\theta) = Cm\cdot\cos m\theta + Dm\cdot\sin m\theta$

$u'' + (2m+1)/x\cdot u' + (r_2+r_1)^2\{k^2 - k/a\cdot W_Z(x)\}\cdot u = 0$

$u = C_0 x^\rho + C_1 x^{\rho+1} + C_2 x^{\rho+2} + \cdots\cdots\cdots$(Frobeniusの方法)

$\therefore\ R = A_{m'n}\cdot x^m[u]_{\rho=0} + B_{m'n}x^m[\frac{\partial u}{\partial \rho}]_{\rho=-2m}$

$b_{i,m'n}(x) = (r_2+r_1)^2A_0kx^2/\{4a(m+1)\} + (r_2+r_1)^2A_1kx^3/\{3a(2m+3)\} + \cdots\cdots\cdots.$

$g_{i,m'n}(x) = \ln x\cdot[(r_2+r_1)^2kA_0x^2/\{2a(2-2m)\} + (r_2+r_1)^2A_1kx^3/\{3a(3-2m)\} + \cdots\cdots]$

$+ (r_2+r_1)^2kA_0(2m-4)x^2/2^2a(2-2m)^2\} + (r_2+r_1)^2kA_1(2m-6)x^3/\{3^2a(3-2m)^2\} + \cdots\cdots$

条件(2)から $i\sin n\pi z/L$ の項が残り(3), (4)をFourier級数に展開してAC, AD, BC, BDを求めると次の解を得る。　$(\varepsilon_0=1, \varepsilon_2=\varepsilon_3=\cdots\cdots\cdots=2)$

$$T=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{\sin(n\pi z/L)}{b_{i,m,n}(r_1)\cdot g_{i,m,n}(r_2) - b_{i,m,n}(r_2)\cdot g_{i,m,n}(r_1)}\times\Big[\Big\{g_{i,m,n}(r_2)\varepsilon_m/(L\pi)\int_0^{2\pi}$$

$$\int_0^L f_1(\varphi,\lambda)\cos m\varphi\cdot\sin(n\pi\lambda/L)d\varphi d\lambda - g_{i,m,n}(r_1)\varepsilon_m/(L\pi)\int_0^{2\pi}\int_0^L f_2(\varphi,\lambda)\cos m\varphi\cdot\sin(n\pi\lambda/L)$$

$$d\varphi d\lambda\Big\}\times b_{i,m,n}(r)\cdot\cos m\theta + \Big\{b_{i,m,n}(r_1)\varepsilon_m/(L\pi)\times\int_0^{2\pi}\int_0^L f_2(\varphi,\lambda)\cos m\varphi\cdot\sin(n\pi\lambda/L)d\varphi d\lambda$$

$$- b_{i,m,n}(r_2)\varepsilon_m/(L\pi)\int_0^{2\pi}\int_0^L f_1(\varphi,\lambda)\cos m\varphi\cdot\sin(n\pi\lambda/L)d\varphi d\lambda\Big\}\times g_{i,m,n}(r)\cos m\theta$$

$$+\Big\{g_{i,m,n}(r_2)\varepsilon_m/(L\pi)\times\int_0^{2\pi}\int_0^L f_1(\varphi,\lambda)\sin m\varphi\sin(n\pi\lambda/L)d\varphi d\lambda$$

$$- g_{i,m,n}(r_1)\varepsilon_m/(L\pi)\int_0^{2\pi}\int_0^L f_2(\varphi,\lambda)\sin m\varphi\cdot\sin(n\pi\lambda/L)d\varphi d\lambda\Big\}\times b_{i,m,n}(r)\sin m\theta$$

$$+\Big\{b_{i,m,n}(r_1)\varepsilon_m/(L\pi)\times\int_0^{2\pi}\int_0^L f_2(\varphi,\lambda)\sin m\varphi\sin(n\pi\lambda/L)d\varphi d\lambda - b_{i,m,n}(r_2)\varepsilon_m/(L\pi)$$

$$\times\int_0^{2\pi}\int_0^L f_1(\varphi,\lambda)\sin m\varphi\sin(n\pi\lambda/L)d\varphi d\lambda\Big\} g_{i,m,n}(r)\sin m\theta\Big].$$

終りに御助言を賜つた松山英太郎博士に深甚の謝意を表す。