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Analysis of Abrupt Discontinuities in Weakly Guiding Waveguides by a Modified Beam Propagation Method

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SUMMARY The beam propagation method (*BPM*) is a powerful and manageable method for the analysis of wave propagation along weakly guiding optical waveguides. However, the effects of reflected waves are not considered in the original *BPM*. In this paper, we propose two simple modifications of the *BPM* to make it relevant in characterizing abrupt discontinuities in weakly guiding waveguides at which a significant amount of reflection is expected to be observed. Validity of the present modifications is confirmed by the numerical results for abrupt discontinuities in step-index slab waveguides and butt-joints between different slab waveguides.

key words: *beam propagation method, reflection, waveguide discontinuity, overlap integral, weakly guiding structure*

1. Introduction

In the design consideration of optical systems it is of fundamental importance to understand the behavior of optical waves propagating along various kinds of waveguides. The beam propagation method (*BPM*) proposed by Feit and Fleck [1] has been successfully applied to the analysis of a wide variety of weakly guiding structures which contain gradual variations in waveguide parameters along the propagation direction. However, actual waveguiding structures contain abrupt discontinuities of the parameters in many places at which an appreciable amount of incident power would be reflected. If one intends to apply the *BPM* to waveguide problems which include such discontinuities, some appropriate modification should be made in order to obtain trustworthy results [2]–[5], since reflected waves are not taken into consideration in the original *BPM* concept.

In the present paper, we modify the *BPM* to make it applicable to the characterization of waveguide discontinuities or butt-joints where the waveguide parameters, such as the waveguide thickness and refractive indices, abruptly change by a significant amount [6]. Variation in the waveguide thickness implies the axial displacement between waveguides interconnected. Two methods of modification proposed in this paper take the Fresnel reflection into consideration. The main algorithm in the *BPM* repeats the same numerical procedure of the Fourier transformation of the modal field and its inverse transformation. One of the two modifications is dealt with in the spatial domain and the other is treated in the spectral domain. Both modifications can easily be incorporated with original *BPM* computing programs. Numerical results for an abrupt discontinuity in film thickness of a step-index slab waveguide are compared with those given by the accurate analytical method [7] and the overlap integral of modal profiles [8]–[11]. Some numerical results are presented for butt-joints between different slab waveguides, which are also compared with the results obtained by the overlap integral [8]–[11]. These numerical results are in good agreement, which verifies that the present methods are applicable to the analysis of abrupt discontinuities or butt-joints of slab waveguides.

2. Analytical Method

Suppose an incoming slab waveguide is connected, with the axial displacement Δs, to an outgoing one having a different film thickness or refractive indices, as shown in Fig. 1. The profile of the refractive index may be steplike or graded in both waveguides. Discontinuities in the refractive index at the interface, for instance $|n_{o1} - n_{o2}|$, may be considerably large. It is, however, assumed that both waveguides satisfy the weakly guiding condition individually. Then the original *BPM* analysis is applicable to each one of the waveguides. Therefore, if the effects of the Fresnel reflection at the interface are taken into account reasonably, the behavior of optical waves

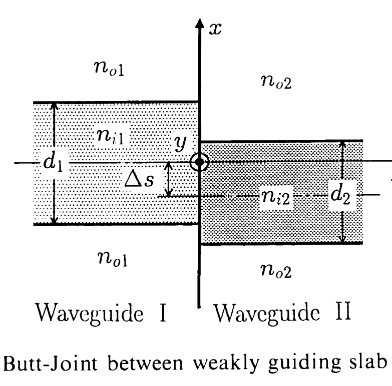

Fig. 1 Butt-Joint between weakly guiding slab waveguides.

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transmitted through and reflected at the junction can be analyzed correctly on the basis of the *BPM* concept.

2.1 Method I (in the spatial domain)

The optical wave in the incoming waveguide can be traced up to the junction by means of the *BPM*, as mentioned above. The field distribution obtained just in front of the junction is the modal field of a normal mode in the incoming waveguide if the single mode excitation is achieved. Since the maximum difference of the refractive index in the outgoing waveguide is very small, the modal profile would not be deformed significantly during passing through the junction. Therefore, the field profile just behind the junction can be obtained by multiplying the incident field by the transmission coefficient

$$t = \frac{2\sqrt{N_1 N_2}}{N_1 + N_2}. \tag{1}$$

The term, "field", in the present paper means the square root of the power flow density. The above-mentioned statement is an intuitive assumption which needs verifying through the numerical check. We define N_1 and N_2 in the above equation as

$$N_j^2 = \frac{\int (n_j F_1)^2 ds}{\int (F_1)^2 ds}, \quad j = 1,\ 2 \tag{2}$$

where F_1 is the field profile in a transverse section of the incoming waveguide and n_j represents the index distribution. The surface integrals must be carried out over the whole cross section. For N_1, (2) can be regarded as an approximation to the scalar variational expression for the propagation constant. Resuming the *BPM* analysis by taking the resultant field as an initial field, the optical waves in the outgoing waveguide can be traced.

2.2 Method II (in the spectral domain)

The *BPM* conceptually replaces a weakly guiding waveguide by a sequence of thin lenses located at equal spaces in a homogeneous material having a proper reference refractive index. Therefor, the waveguide butt-joint shown in Fig. 1 can be regarded as an interface between different dielectric materials containing different sequences of thin lenses. The basic algorithm of the *BPM* is a consecutive procedure of plane-wave expansions of the modal field by means of the Fourier transformation and reconstructions of it by the inverse transformation along the lens sequence [1]. At the interface between the two waveguides each expanded plane wave is partly transmitted with the transmission coefficient [12]

$$t' = \frac{2\sqrt{n_{r1}\cos\alpha\sqrt{n_{r2}^2 - n_{r1}^2\sin^2\alpha}}}{n_{r1}\cos\alpha + \sqrt{n_{r2}^2 - n_{r1}^2\sin^2\alpha}}, \tag{3}$$

where α is the incident angle measured from the normal of the interface and takes a discrete value which is determined by the discrete Fourier transformation. The reference refractive indices of the incoming and outgoing waveguides are denoted by n_{r1} and n_{r2}, respectively, which usually take the refractive index of the substrate of each waveguide as in the present paper. For a weakly guiding waveguide, the reference refractive index may take such a different value as the refractive index of the film or some intermediate value [1]–[4].

This coefficient is valid for *TE* waves in slab waveguides, since the boundary conditions at the interface are automatically satisfied by the discrete Fourier transformation in both the incoming and outgoing waveguides. For analyzing the *TM* mode we should use its counterpart.

Recently, reflection operators have been proposed for the *BPM* analysis and mainly applied to the reflected waves from abrupt discontinuities [2]–[4]. These operators contain the space-dependent reflection coefficient which is defined by the difference between the space distributions of the refractive index on both sides of the discontinuity. However, this coefficient also seems to be rather an intuitive approximation.

Dominant components of the plane waves constructing the modal field in any type of weakly guiding structure are incident almost normally upon the interface. Transmission and reflection coefficients of these plane waves are closely identical for both polarizations. This means that the transmission coefficient (3) is applicable to an arbitrarily polarized plane wave having a predominant expansion coefficient without causing any appreciable errors. Consequently, multiplying the amplitude of each plane-wave component in the incoming waveguide by t' at the interface and converting the propagation constant of the plane-waves into $n_{o2}k_0$ in the outgoing waveguide, we can estimate the field distribution at any place in the outgoing waveguide.

Both modifications of the *BPM* in this paper would be expected to give correct results for a discontinuity problem where an air gap exists between the waveguides connected to each other, as long as the discontinuity in the refractive index is not extremely large.

3. Numerical Results

3.1 Step Discontinuities

To verify the present modifications of the *BPM*, at first, we apply them to a step discontinuity in film thickness of a symmetric slab waveguide. It is assumed in Fig. 1 that the ratio of the film thickness on both sides of the

Table 1 Normalized transmitted power of the guided TE waves at the step discontinuity as a function of $k_0 d_2$. The waveguide parameters are chosen so that $d_2/d_1 = 0.5$, $n_{o1} = n_{o2} = 1.0$, and $n_{i1} = n_{i2} = 1.01$.

$k_0 d_2 = k_0 d_1/2$		10.0	20.0	40.0
P_T	Reference [7]	0.990	0.957	0.863
	Eq.(4)	0.990	0.957	0.863
	Method I	0.996	0.969	0.865
	Method II	0.996	0.969	0.865

discontinuity is $d_2/d_1 = 0.5$ and that the refractive indices of the film and substrate are $n_{o1} = n_{o2} = 1.0$ and $n_{i1} = n_{i2} = 1.01$, respectively.

In this case, an accurate analytical solution has been given by Morita [7]. In order to estimate the transmitted power of the guided TE waves in weakly guiding waveguides it is considerably a simple way to apply the following overlap integral of the modal profiles:

$$P_T = \frac{4\beta_1\beta_2}{(\beta_1+\beta_2)^2} \cdot \frac{\left\{\int F_1 \cdot F_2^* \, dx\right\}^2}{\int |F_1|^2 dx \cdot \int |F_2|^2 dx} \quad (4)$$

where F_i and β_i ($i = 1$ *or* 2) are the field distribution and the propagation constant of the normal mode under consideration, respectively, and the asterisk indicates complex conjugation [8]–[11].

Table 1 summarizes the transmitted power of the guided TE waves estimated by Method I and II as a function of $k_0 d_2$, where k_0 is a free-space wavenumber. For the sake of comparison, the results given by Morita [7] and the overlap integral (4) are also listed in this table. The results obtained by Method I and II are in good agreement with those by the accurate methods: relative differences between them are less than only 2%.

3.2 Waveguide Butt-Joints

We also apply our methods to butt-joints between slab waveguides with the axial displacement Δs. We consider two kinds of index distribution for the outgoing waveguide: one is of step-index type and the other is of square-law-index type. In each case, the incoming waveguide has a step-index profile and the refractive indices are chosen so that $n_{o1} = 1.0$, $n_{i1} = 1.01$. The film thicknesses of the incoming and outgoing waveguides are assumed as $d_1 = 3\lambda$ and $d_2 = 2\lambda$, respectively, where λ is a free-space wavelength. The multi-layer approximation is applied to the modal analysis in square-law-index waveguides [13].

3.2.1 Step-Index Profile

First, we consider the outgoing waveguide of step-index type. The refractive indices of the film and substrate regions are assumed as $n_{o2} = 1.99$ and $n_{i2} = 2.0$, respectively. Both the incoming and outgoing waveguides with these parameters support only the dominant mode. Figure 2 shows the power-coupling efficiency of the structure for the TE guided waves as a function of the axial displacement.

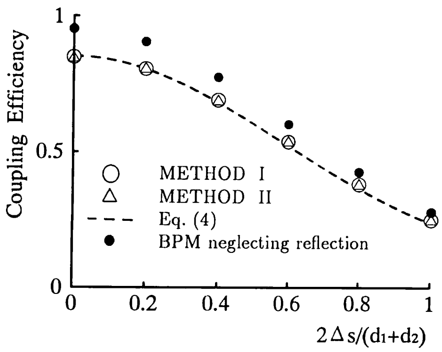

Fig. 2 Normalized transmitted power of the guided TE waves at the butt-joint between step-index slab waveguides as a function of normalized axial displacement. The waveguide parameters are chosen so that $d_1 = 3\lambda$, $d_2 = 2\lambda$, $n_{o1} = 1.0$, $n_{i1} = 1.01$, $n_{o2} = 1.99$, and $n_{i2} = 2.0$.

The ordinate is the transmitted power normalized relative to the input power. The abscissa represents the normalized displacement $2\Delta s/(d_1+d_2)$ between the waveguides. Open circles show the results by Method I and triangles represent those by Method II. The numerical results obtained by the overlap integral (4) are also shown in the figure by the dashed line.

As shown in the figure, the results obtained by Method I and II are in excellent agreement, differing from the dashed line only by 2% at most. The results obtained with the original *BPM* without considering the Fresnel reflection are also shown in this figure and the maximum error is more than 10%.

The method of overlap integral is not only accurate, as shown in Table 1, but also convenient for calculating the transmitted power, from which the radiated power can also be estimated approximately. In addition to the transmitted power, the field distribution including both guided and radiated waves is another important feature for our deep understanding of wave propagation. The overlap integral method, however, does not offer any information on interference patterns which depend strongly on the refractive index distribution and the axial displacement. On the contrary, one of the great advantages of the *BPM* is easiness in calculating the field distribution. Upon neglecting the reflected waves in Method II, we obtain the field distribution along the butt-jointed slab waveguides which are illustrated in Fig. 3, where the normalized axial displacement $2\Delta s/(d_1+d_2) = 0.6$. We have obtained almost the same result with Method I, which is not shown in this paper. This illustration is a great help to our visual understanding on the modal conversion at the dis-

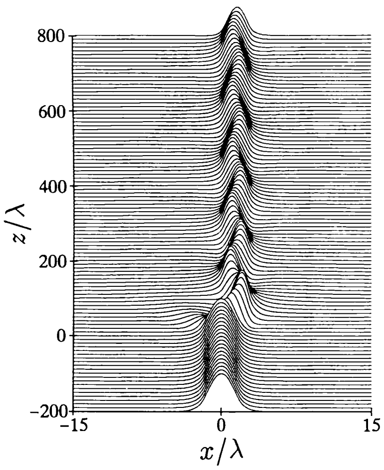

Fig. 3 Field distribution along the butt-jointed waveguides obtained by Method II, where the outgoing waveguide is of step-index type and the normalized axial displacement $2\Delta s/(d_1 + d_2) = 0.6$. The waveguide parameters are the same as Fig. 2.

continuity which causes the beam undulation of optical waves through a fairly long range behind the junction.

3.2.2 Square-Law-Index Profile

Next, we consider the outgoing waveguide of square-law-index type with the distribution expressed as

$$n_2(x) = \begin{cases} n_{o2}, & x < -d_2/2 \\ n_{i2}\left\{1 - \delta n(2x/d_2)^2\right\}, & |x| \leqq d_2/2 \\ n_{o2}, & x > d_2/2 \end{cases} \tag{5}$$

where $n_{o2} = 1.99$, $n_{i2} = 2.0$, and $\delta n = 0.005$. The outgoing waveguide supports only the dominant mode also in this case. Figure 4 shows the power-coupling efficiency of the structure for the guided *TE* waves as a function of the axial displacement. The ordinate and abscissa are the same as Fig. 2. We can recognize good agreement in the results by Method I and II. The discrepancies from the dashed line which illustrates the results given by (4) are about 2% also in this case. The results obtained with the original *BPM* without any modification contain as many errors as those in Fig. 2.

The field distribution obtained by Method II along the butt-jointed slab waveguides is shown in Fig. 5, where $2\Delta s/(d_1 + d_2) = 0.6$ and the reflected waves are also neglected. The beam undulation of optical waves rapidly fades away behind the junction in this case. This smooth transition to the guided mode is due to self-focusing effects of the square-law-index distribution.

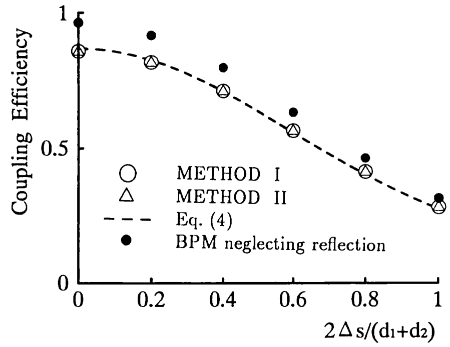

Fig. 4 Normalized transmitted power of the guided *TE* waves at the butt-joint between a step-index slab waveguide and a square-law-index slab waveguide as a function of normalized axial displacement, where $\delta n = 0.005$ and the other parameters are the same as Fig. 2.

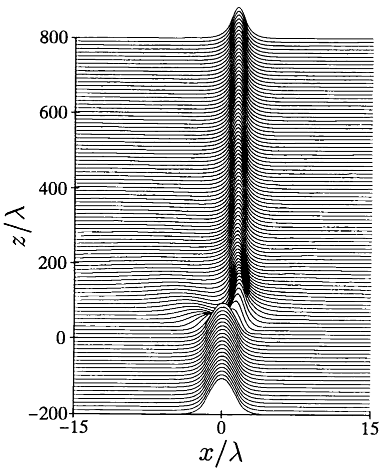

Fig. 5 Field distribution along the butt-jointed waveguides obtained by Method II, where the outgoing waveguide is of square-law-index type and the normalized axial displacement $2\Delta s/(d_1 + d_2) = 0.6$. The waveguide parameters are the same as Fig. 4.

3.2.3 Discussion on More Strongly Guiding Waveguides

It has been shown that the two methods presented here equally offer good results in the analysis of butt-joints between weakly guiding waveguides. It is, however, expected that errors included in the results increase as the index difference $\Delta n (= n_{i2} - n_{o2})$ of the outgoing waveguide becomes large. In order to discuss this issue we deal with butt-joints to which outgoing waveguides with larger $\Delta n/n_{o2}$ values are connected.

Table 2 shows the power-coupling efficiency for the guided *TE* waves as a function of the axial displacement. The refractive indices of the outgoing waveguide, which is assumed to have a step-index profile, are chosen as $n_{o2} = 1.99$, $\Delta n/n_{o2} = 0.005$, 0.05, or 0.1. Table 2 also includes the results obtained with the overlap integral (4) whose validity for larger $\Delta n/n_{o2}$

Table 2 Normalized transmitted power of the guided *TE* waves at the butt-joint between step-index slab waveguides as a function of normalized axial displacement. Except for n_{i2}, the waveguide parameters are the same as Fig.2.

Δs		0.0	0.2	0.4	0.6	0.8	1.0
$\frac{\Delta n}{n_{o2}} = 0.005$	Method I	0.849	0.808	0.692	0.539	0.382	0.250
	Method II	0.844	0.805	0.688	0.537	0.379	0.249
	Eq.(4)	0.852	0.806	0.683	0.523	0.366	0.238
$\frac{\Delta n}{n_{o2}} = 0.05$	Method I	0.581	0.507	0.438	0.282	0.184	0.089
	Method II	0.584	0.502	0.439	0.276	0.182	0.085
	Eq.(4)	0.581	0.535	0.417	0.276	0.160	0.084
$\frac{\Delta n}{n_{o2}} = 0.1$	Method I	0.544	0.475	0.410	0.265	0.171	0.084
	Method II	0.582	0.418	0.440	0.221	0.186	0.062
	Eq.(4)	0.528	0.486	0.376	0.245	0.139	0.072

values has been confirmed through a comparison with two other accurate methods [7],[14]. Relative differences in the numerical results are less than 2% for butt-joints studied in each paper. One glance at Table 2 tells that both of Method I and II fail to be valid for $\Delta n/n_{o2} = 0.1$. Apart from this, readers are reminded that the *BPM* itself starts to be unstable for $\Delta n/n_{o2}$ above 0.05. Judging from these facts, we can trust both methods equally to offer fairly accurate results up to around $\Delta n/n_{o2} = 0.05$.

4. Conclusions

In the present paper, two methods of modification to the *BPM* are made for the application of it to the analysis of discontinuities in weakly guiding structures. The modification is essentially based on the Fresnel reflection at the interface. One is introduced in the spatial domain and the other in the spectral domain. It is clarified numerically that these methods are applicable with satisfactory accuracy to the analysis of abrupt discontinuities or butt-joints of weakly guiding slab waveguides. The present methods can easily be extended to the analysis of a waveguide butt-joint which contains an air gap at the junction.

Acknowledgments

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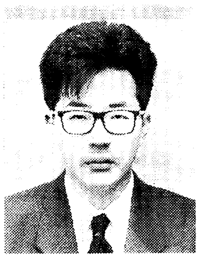

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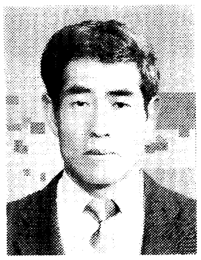

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