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Study on the Front of a Turbidity Current

by

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Abstract

In this paper the author considers the flow property of the head region of a turbidity current, a kind of a density current. In the experiments it was observed that the ambient fluid is entrained into a moving denser fluid at the joint between the head and the following flow. The growth rate of the maximum thickness of the head is almost constant in one run, and it increases as the slope of the bed becomes steeper. The phenomenon is treated as unsteady two-dimensional flow, and the two layers model is adopted. Governing equations of the thickness of a density current, extra density (or turbidity) and the velocity of the head are developed by use of the characteristic curve method, and the characteristic equations are integrated in the case when the growth rate of maximum thickness of the head, entrainment coefficient etc. are constant. The solutions show good agreement with experimental data.

1. Introduction

When the turbid water flows into a reservoir, a turbidity current takes place because of the density difference between the turbid water and the reservoir water. This flow is also called a bottom density current. There is a considerable interest in turbidity currents because they impair water quality and reduce storage capacity in reservoirs by suspension and deposition of suspended solids which form the turbidity currents. In that case, the motion of the front is as important and interesting as the steady following flow.

From the past, several investigators have commented on the characteristic shape of the head of density currents. For example, von Kārmān[2] made a preliminary theoretical analysis of the shape for invisid fluids and Prandtl[3] discussed the shape and the velocity of density current heads in general terms. Ippen and Harleman[4] suggested that the shape of the head remained constant, if expressed in terms of maximum thickness.

Attemps on the theoretical treatment for the motion of a gravity current head were also made. Plapp and Mitchell[5] investigated the turbidity current theoretically on the basis of boundary layer assumption. Their equation of motion is the one for steady state, and the physical quantities such as the thickness, velocity and the density difference are assumed to change exponentially with moved distance. It is a question whether they knew the existence of the characteristic head of a density current. Britter and Simpson[6] considered the case in which the head of a gravity current advancing along the horizontal

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* A part of this study is to be published in literature[1] in Japanese.

bottom was brought to rest by an opposing flow. But the motive force of the current is different in the case of horizontal bottom and in that of inclined bottom. In the case of inclined bottom the force due to gravity becomes important. Fukuoka et al.[7] treated analytically the motion of the head of inclined density current as unsteady two-dimensional problem, and found that the maximum thickness increases with moved distance. They failed to take into account the entrainment of ambient fluid which should not be ignored. The motive force of a density current comes from density difference between the two fluids. It is considered that their value of drag coefficient which seems too large would have come from this neglect of entrainment.

This paper considers the motion of the turbidity current head experimentally and theoretically. Experiments were performed in a two-dimensional tank whose bottom is inclined. In the analysis, it is assumed that the flow is unsteady two-dimensional and that the density current flows down a slope entraining the ambient fluid. The pick up or deposition of the particles at the bed surface is not considered.

2. The experiments

To study the motion of the head of a turbidity current, some of experiments were performed in a laboratory flume which was made of transparent plastic on one side, and had a sloping bottom in it and a weir at the downstream end. Figure 1 shows the flume. The slope of bed can be changed as desired. Density difference was set up with either salt solution or pearl clay suspension.

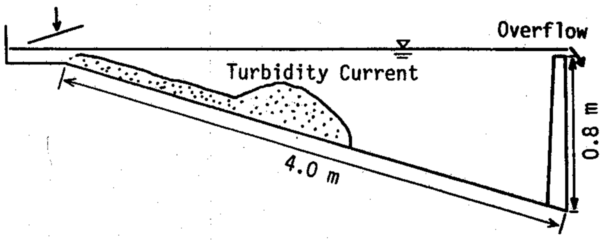

Fig. 1 The experimental flume

To run an experiment, after a slope, density difference and discharge of a denser fluid were set up to desired values, the inlet valve was opened. During the experiments, the denser fluid which was marked with a white dye flowed down a slope and the fresh water flowed out over a weir at the down stream end. The shape of the head was investigated by tracing the outline of the head from a video tape recorder (V. T. R.) monitor. The density was found by withdrawing the samples from moved front in a moment with fine copper tubes and measuring electric conductivity. The copper tubes were connected with a fairly evacuated bottle. The velocity of the front was measured by timing the passage of the head from a V. T. R. monitor. Motion of the water around the head was studied in the bead-suspension experiments by tracing the movement of individual bead from frame to frame of a series of photographs taken regularly.

3. The Experimental Results

3. 1 The Shape of the Head

The photograph of a density current observed in the experiment is shown in Fig. 2.

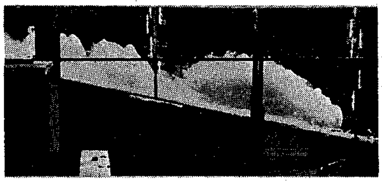

Fig. 2 A photograph of a turbidity current

When the flow is started, a rounded nose moved through static water usually overhanging its base slightly and increasing its thickness with a flow down. The shape of the front was obtained at certain position along the flume. The pictures were projected in a V. T. R. monitor and traced directly. Figure 3 shows the dimensionless shapes obtained in the experiments with either salt solution or pearl clay suspension. It can be seen that the shape is intermediate between that obtained by Ippen and Harleman[4] and that obtained by Keulegan[5]. Within the limit of experiments, the difference of the shape with experimental condition is not clear. It seems that irregularity of the shape has come out from periodic expansion and contraction of the head region in a flow down.

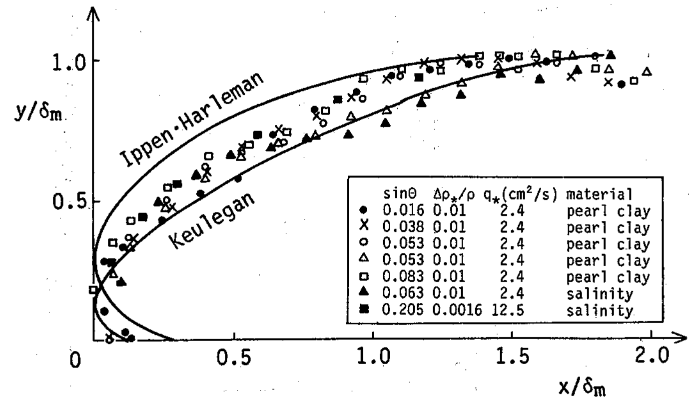

Fig. 3 The shape of the density current head

3. 2 The Motion of ambient Fluid

One of the flow patterns of the water around the head obtained in the bead-suspension experiments is shown in Fig. 4. There can be seen remarkable entrainment of ambient fluid at the constricted part of the moving fluid where the head joins with the steady following flow. The shape of the head is similar to an ellipse. If we obtain the motion of water by means of velocity potential theory in the case when an ellipic cylinder

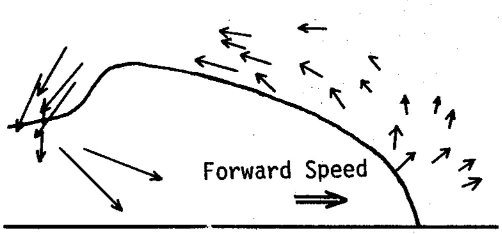

Fig. 4 Velocity vectors of ambient fluid near the head

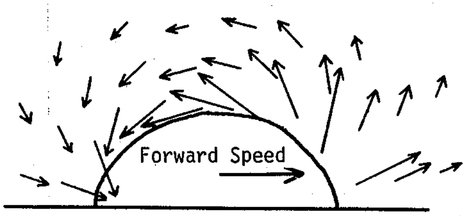

Fig. 5 Velocity vectors of ambient fluid induced by the motion of an ellipic cylinder obtained by means of velocity potential theory

moves in still water, the flow pattern becomes as in Fig. 5. By comparing the two, it is found that the flows are similar manners except at the constricted part of moving fluid. Thus it seems that the most part of dillusion is induced by the entrainment at the constricted part.

3. 3 Salinity Distribution

The salinity distribution at various parts in the head region of a density current has been obtained by Fukuoka et al.[7]. Here, the author measured the salinity distribution in the section of maximum thickness when the front reached some points along the flume. Figure 6 shows one of the results, in which C denotes the salinity. It is found that the denser fluid is fairly dilluted in the upper part of the head and that the salinity in the head decreases with a flow down. The change in the mean salinity over the thickness of the head with a flow down is shown in Fig. 7 in terms of extra density, where $\Delta\rho_0$ is the value of $\Delta\rho$ when the front reaches the position $x=x_0$. It is clear that there exists a fair amount of entrainment of ambient fluid.

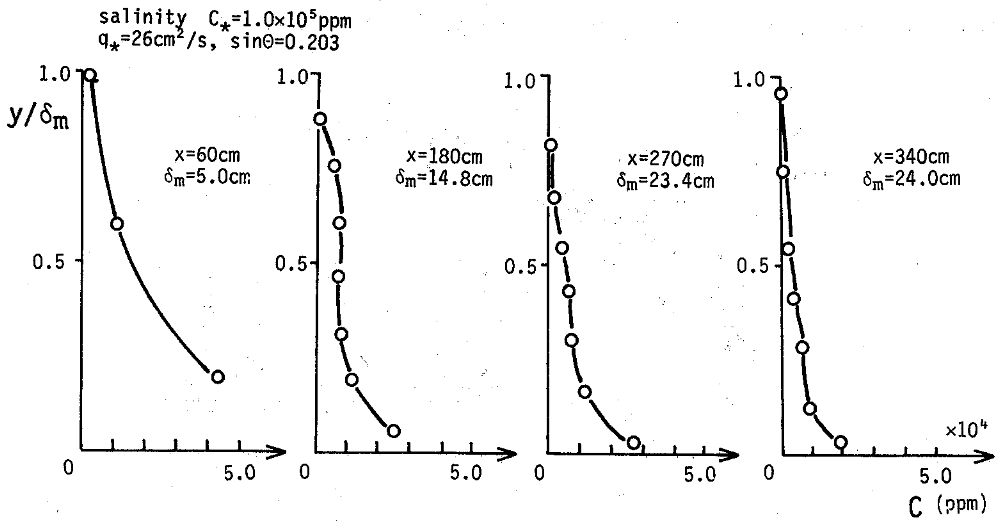

Fig. 6 Salinity distribution measured at the section of maximum thickness

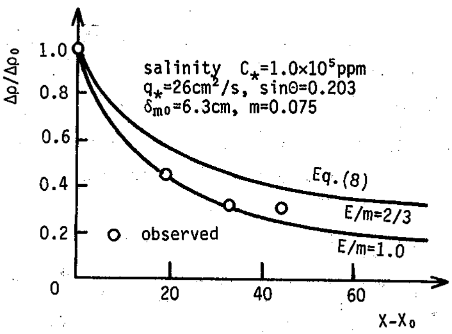

Fig. 7 Change in the mean salinity (extra density) along the thickness of the head

3. 4 The Growth of the Head

As was mentioned before, the thickness of the head increases with a flow down.

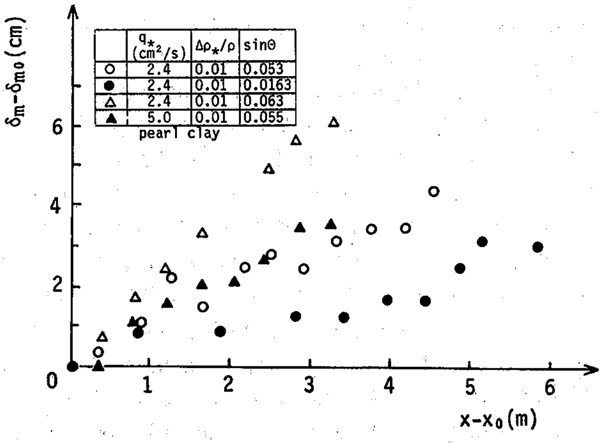

Fig. 8 Change in the maximum thickness of the head

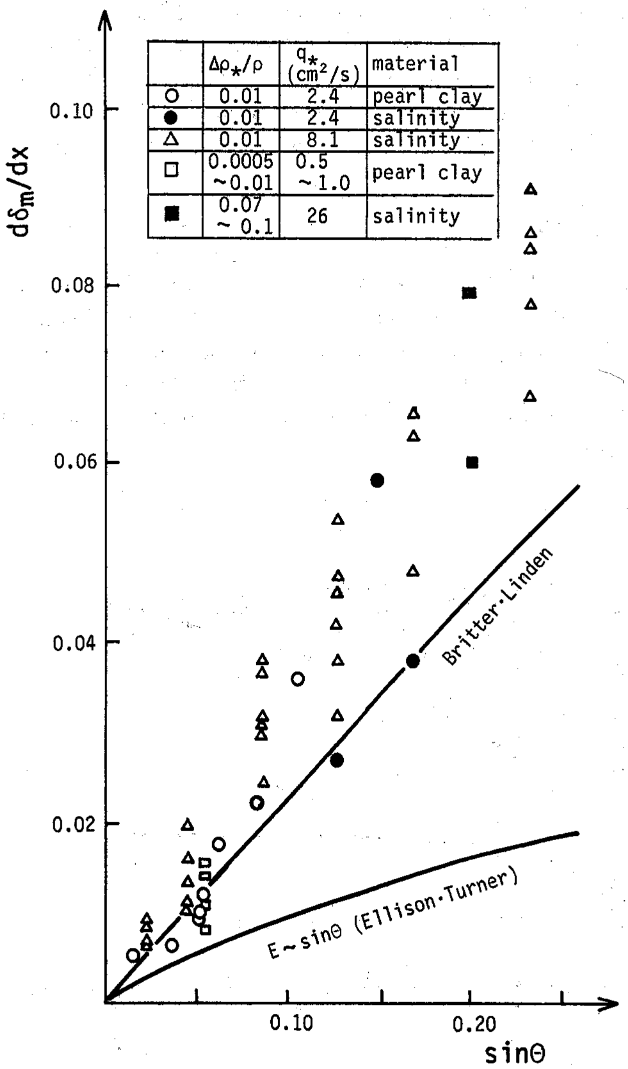

Fig. 9 The growth rate of maximum thickness of the head versus the slope of the bed

Figure 8 shows the change in maximum thickness of the head δ_m with moved distance. In Fig. 8 x denotes the position of the front and δ_{m0} denotes the value of δ_m when the front reaches the position $x=x_0$. It is shown that the density current head increases its maximum thickness almost liniearly with moved distance, as was shown by Fukuoka et al.. It is also shown that the growth rate of maximum thickness $d\delta_m/dx$ changes with the experimental conditions. To examine this, experiments were performed, in which the slope of bed, the rate of emission of denser fluid and density difference were changed. As the results of the examinations, the effect of the discharge of denser fluid or that of initial density difference on $d\delta_m/dx$ are not clear within the limits of these experiments, but the effect of the slope of bed is clear. Figure 9 shows the circumstances well. It is shown that as the slope becomes steeper the growth rate of maximum thickness becomes larger. It is interesting that the variation of $d\delta_m/dx$ with bed slope shows a similar tendency to the result for entrainment coefficient obtained in the steady flow by Ellison and Turner[9]. The values of $d\delta_m/dx$ are plotted above the curve of their entrainment coefficient. As the cause of this gap, it is considered that the value of entrainment coefficient itself should be larger than that in the steady following flow because the entrainment takes place as in Fig. 4 and that the head may spread not only by entrainment but also by the velocity difference between the head and the steady following flow. Though this feature was also obtained recently by Britter and Linden[10], their curve of $d\delta_m/dx$ comes below the present data.

3.5 The Velocity of the Head

The velocity of the front was plotted versus the position in Fig. 10, where U and X are nondimensional quantities of the velocity and the moved distance normalized by means of the values of velocity and the maximum thickness when the front reaches the position $x=x_0$. It is shown that the velocity of the head increases at first and gradually approaches to a constant value which seems to depend on the slope of bed and the rate of discharge of extra weight.

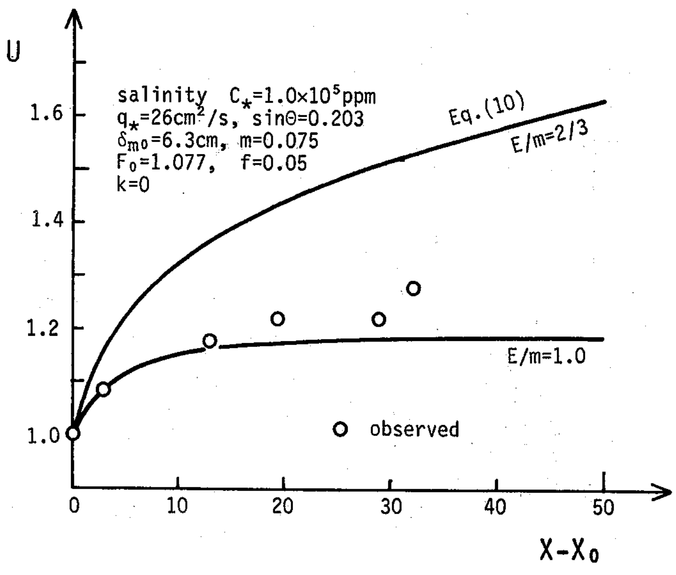

Fig. 10 Change in the velocity of the head

4. Theory

Let us consider the two-dimensional unsteady state of a layer flowing down a sloping bottom under a stationary ambient fluid. Figure 11 shows the simplified model for analysis. The two layers model is adopted, and it is assumed that the denser fluid flows down a slope entraining the ambient fluid. Basic equations are those of conservation of volume and extra density of the denser fluid and the x-component of equation of motion. They are as follows;

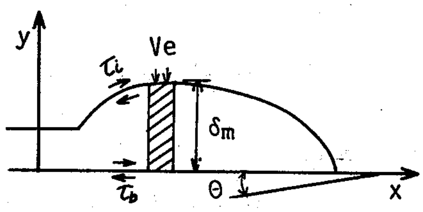

Fig. 11 Simplified model for analysis

$$\frac{\partial \delta}{\partial t}+\frac{\partial q}{\partial x}=V_e \tag{1}$$

$$\frac{\partial \Delta\rho}{\partial t}+\frac{q}{\delta}\frac{\partial \Delta\rho}{\partial x}=-\frac{\Delta\rho V_e}{\delta} \tag{2}$$

$$\int_0^\delta \rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)dy=-\int_0^\delta \frac{\partial p}{\partial x}dy+\int_0^\delta \rho g\sin\theta\, dy+\tau_i-\tau_b \tag{3}$$

where, δ is the flow thickness of denser fluid, t is time, x is the distance in a flow direction, y is the distance from the bed, $q=\int_0^\delta u\,dy$, V_e is the velocity of entrainment, ρ is the density of the fluid, $\Delta\rho$ is the density difference between the two fluids, u and v are the velocity components along the x- and y-axes respectively, p is pressure, θ is the angle of the bed slope, g is the acceleration due to gravity, τ_i and τ_b are the interfacial and bed shear stresses respectively. If we assume that the pressure is hydrostatic, the first two of the right hand side of Eq. (3) are rewritten as below.

$$\Delta\rho g\delta\sin\theta-g\cos\theta\frac{\partial}{\partial x}\left(\frac{1}{2}\Delta\rho\delta^2\right)$$

in which, the first term represents the gravity force accelerating the layer and the second term expresses the difference of the integrated hydrostatic pressure on the planes x and $x+dx$. When the slope of bed becomes steeper, the former becomes important.

By employing the relation $q=\bar{u}\delta$, one can change Eq. (1) as Eq. (4) along the characteristic curve $dx/dt=\bar{u}$.

$$\frac{d\delta}{dx}=E-\frac{\delta}{\bar{u}}\frac{\partial \bar{u}}{\partial x} \tag{4}$$

where $\bar{u}$ is the mean value of u along the thickness of the density current and $E=V_e/\bar{u}$ is entrainment coefficient. Equation (4) represents that the density current increses its thickness with entrainment and distortion of flow velocity. Equation (2) is rewritten as Eq. (5) along the same characteristic curve.

$$\frac{d\Delta\rho}{dx} = -\frac{\Delta\rho E}{\delta} \tag{5}$$

Next, the form of left hand side of Eq. (3) can be changed as below by use of continuity equation $\partial u/\partial x + \partial v/\partial y = 0$, $q = \bar{u}\delta$, Boussinesq approximation and Eq. (1).

$$\rho\int_0^\delta\left(\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y}\right)dy$$

$$= \rho\left[\frac{\partial}{\partial t}\int_0^\delta u\,dy + \frac{\partial}{\partial x}\int_0^\delta u^2\,dy - u(\delta)\cdot\left(\frac{\partial\delta}{\partial t} + \frac{\partial}{\partial x}\int_0^\delta u\,dy\right)\right]$$

$$= \rho\left\{\left(\frac{\partial\delta}{\partial t} + \beta\frac{\partial q}{\partial x}\right)\bar{u} + \left(\frac{\partial\bar{u}}{\partial t} + \beta\bar{u}\frac{\partial\bar{u}}{\partial x}\right)\delta - u(\delta)\bar{u}E\right\}$$

where, $u(\delta)$ is the value of u at $y=\delta$ and β is the momentum correction factor; $\int_0^\delta u\,dy = \beta\bar{u}^2\delta$. The above equation is simplified if we assume momentum correction factor to be unity from the experimental results by Fukuoka et al[7]. Further, we introduce drag coefficient defined as $\tau_b - \tau_i = f\rho\bar{u}^2$. thus Eq. (3) is rearranged as follows;

$$\frac{\partial\bar{u}}{\partial t} + \bar{u}\frac{\partial\bar{u}}{\partial x} + \frac{1}{\delta}\left\{E(1-k) + f\right\}\bar{u}^2$$

$$= \frac{\Delta\rho}{\rho}g\sin\theta - \frac{1}{\rho\delta}g\cos\theta\frac{\partial}{\partial x}\left(\frac{1}{2}\Delta\rho\delta^2\right) \tag{6$'$}$$

where, $k = u(\delta)/\bar{u}$. Above equation is rewritten as Eq. (6) along the same characteristic curve.

$$\frac{d\bar{u}^2}{dx} + \frac{2}{\delta}\{E(1-k) + f\}\bar{u}^2$$

$$= \frac{2\Delta\rho}{\rho}g\sin\theta - \frac{2}{\rho\delta}g\cos\theta\frac{\partial}{\partial x}\left(\frac{1}{2}\Delta\rho\delta^2\right) \tag{6}$$

Below we will solve Eqs. (4), (5), and (6) which hold along the above characteristic curve. From the past it has been shown experimentally that the density current head increases its maximum thickness almost linearly with moved distance and that the shape remains similar figures in a flow down. These properties were also obtained in the present experiments. If we assume $m(=E - \delta/\bar{u}\cdot\partial\bar{u}/\partial x)$ to be constant applying these results, the following expression is obtained.

$$\delta = m(x - x_0) + \delta_0 \tag{7}$$

where, δ_0 is the value of δ when the front reaches the position $x = x_0$. Below, subscript "$_0$" will be used to denote the value when the front reaches this point. Next, Eq. (5) is integrated if we employ Eq. (7) and assume E to be constant. The integrated form is as follows;

$$\frac{\Delta\rho}{\Delta\rho_0} = \left(\frac{\delta}{\delta_0}\right)^{-E/m} = \left\{\frac{m}{\delta_0}(x - x_0) + 1\right\}^{-E/m} \tag{8}$$

At last we consider Eq. (6). If we assume that the shape of a head remains similar figures and that the growth rate of maximum thickness of the head is constant, the velocity of the foremost point becomes proportional to that averaged along the section of the maximum thickness. In such a section as $\partial\delta/\partial x$ is equal to zero, the second term of the right hand side of Eq. (6) is negligible, when we assume that the variation of $\Delta\rho$ in a head region is small compared with $\Delta\rho$ itself. Further if we substitute Eqs. (7) and (8) into Eq. (6) and introduce nondimensional quantities: $U=\bar{u}/\bar{u}_0$, $X=x/\delta_0$ and $F_0=\bar{u}_0/\sqrt{\Delta\rho_0 g\delta_0/\rho}$; the following equation is obtained.

$$\frac{dU^2}{dX}+\frac{2\{E(1-k)+f\}}{m(X-X_0)+1}U^2$$
$$=2F_0^{-2}\sin\theta\{m(X-X_0)+1\}^{-\frac{E}{m}} \tag{9}$$

This is the governing equation for the velocity of the head of a turbidity current. If we assume that E, m, k and f are constant, Eq. (9) is integrated as follows;

$$U^2=\frac{2F_0^{-2}\sin\theta}{2f+E(1-2k)+m}\{m(X-X_0)+1\}^{1-\frac{E}{m}}$$
$$+\left\{1-\frac{2F_0^{-2}\sin\theta}{2f+E(1-2k)+m}\right\}\{m(X-X_0)+1\}^{-2\frac{E(1-k)+f}{m}} \tag{10}$$

5. Results and Discussion

In order to examine the applicability of Eqs. (8) and (10) both the calculated and experimental results for salinity (in terms of extra density) and the velocity of the head are plotted in Figs. 7 and 10. In the calculation, for the value of m experimental value of the growth rate of maximum thickness is adopted. Because the ratio of entrainment to the growth of the thickness of a head is not clear, calculation is carried out for $E/m=2/3$ and 1.0. The value of k is put to zero because the velocity becomes zero near the interface, and 0.05 is adopted for the value of f by referring to Blasius' law et al.[11].

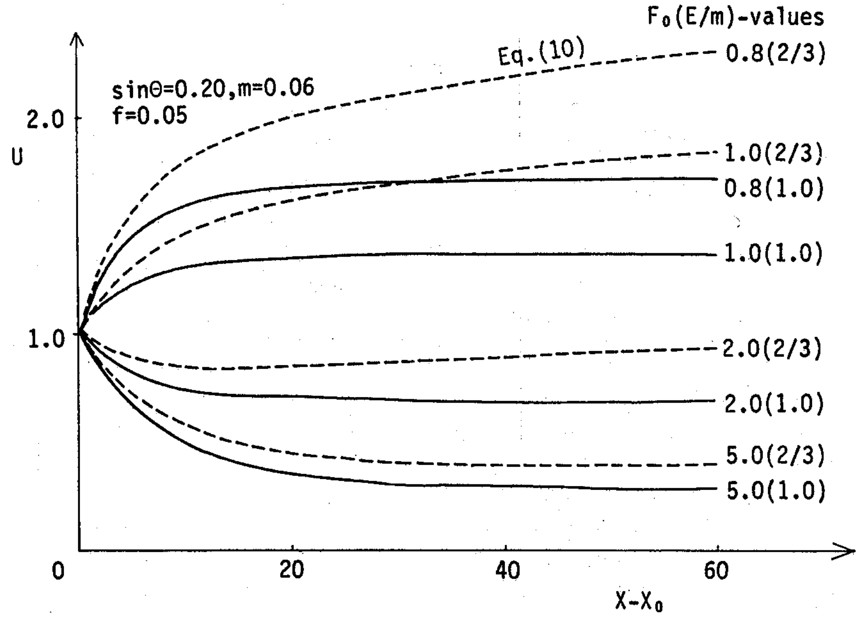

Fig. 12 Effect of the initial value F_0 on the change in the velocity of the head

Figure 7 shows that both experimental and calculated values for salinity have a similar tendency and it is expected that the value of E/m may be the degree of $2/3 \sim 1.0$, though the accuracy of observed value for mean salinity is slightly questionable. As was mentioned before, velocity of the foremost point becomes proportional to the mean velocity at the section of the maximum thickness, if we assume that the shape of the head remains similar figures and that the growth rate of maximum thickness of the head is constant. Thus Eq. (10) also shows a property of the change in the velocity of the foremost point. Figure 10 shows that the calculated value for $E/m=1.0$ gives good agreement with experimental data. Next, the effect of the initial value of densimetric Froude number F_0 on the change in velocity is shown in Fig. 12. It is shown that if the initial value F_0 is small the head is accelerated and in the reverse case velocity is reduced.

6. Conclusion

The summary of the results of this paper is as follows; It was confirmed that the inclined density current flows down a slope spreading its head. The growth rate of maximum thickness of the head remains nearly constant in a flow down and its value changes with a slope of bed. The steeper the slope is, the greater the growth rate is. Entrainment of ambient fluid into a density current head takes place at the constricted part of moving fluid where the head joins with the steady following flow. The theory of unsteady turbidity current along a sloping bottom was developed on the basis of boundary layer assumption, and the solutions have been obtained by means of the characteristic curve method of partial differential equation. Theory shows a good agreement with experimental data.

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References

1) Hirano, M. and K. Hadano: Characteristics of motion of density current head, Proc. of JSCE. 314, 1981 (in Japanese).
2) von Kármán, T.: The engineer grapples with non-linear problem, Bull. Am. Math. Soc. 46, 615, 1940.
3) Prandtl, L.: Essentials of fluid dynamics, Blackie and Son, Ltd. Grasgow, 1952.
4) Ippen, A.T. and D.R.F. Harleman: Steady state characteristics of subsurface flow, U.S. Natl. Bur. Standards Cir. 521, 79, 1952.
5) Plapp, J.E. and J.P. Mitchell: A hydraulic theory of turbidity currents, J. Geophy. Res. 65, 3, 1960.
6) Britter, R.E. and J.E. Simpson: Experiments on the dynamics of a gravity current head, J. Fluid Mech. Vol. 88, Part 2, 1978.
7) Fukuoka, S., K. Mizumura and T. Kano: Fundamental study on the dynamics of the head of gravity currents, Proc. of JSCE. 274, 1978 (in Japanese).
8) Keulegan, G.H.: Twelfth progress report on model laws for density currents. The motion of saline fronts in still water, U.S. Natl. Bur. Standards Rep. 5831, 1958.
9) Ellison, T.H. and J.S. Turner: Turbulent entrainment in stratified flows, J. Fluid Mech. Vol. 6, 423, 1959.
10) Britter, R.E. and P.F. Linden: The motion of the front of a gravity current travelling

down an incline, J. Fluid Mech. Vol. 99, Part 3, 1980.
11) JSCE: The formulae of hydraulics, 1971 (in Japanese).

List of Simbols

C salinity
C_* salinity of the salt solution before supply
E entrainment coefficient, $E=V_e/\bar{u}$.
F_0 value of densimetric Froude number when the front reaches the position $x=x_0, F_0=\bar{u}_0/\sqrt{\Delta\rho_0 g\delta_0/\rho}$.
f drag coefficient, $\tau_b-\tau_i=f\rho\bar{u}^2$.
g acceleration due to gravity
m growth rate of the thickness of a density current
p pressure
q flow rate of a density current per unit width
q_* rate of supply of denser fluid per unit width
t time
U normalized velocity, $U=\bar{u}/\bar{u}_0$.
u x-component of the velocity
$\bar{u}$ mean value of u averaged over the flow thickness of denser fluid
$\bar{u}_0$ value of $\bar{u}$ when the front reaches the position $x=x_0$
V_e velocity of entrainment into a density current
v y-component of the velocity
X normalized distance, $X=x/\delta_0$
X_0 value of X corresponding to the position $x=x_0$
x distance taken to a flow direction
y distance taken above from the bed
β momentum correction factor, $\int_0^\delta u^2 dy=\beta\bar{u}^2\delta$
δ flow thickness of the density current
δ_0 value of δ when the front reaches the position $x=x_0$
δ_m maximum thickness of the head of a density current
δ_{m0} value of δ_m when the front reaches the position $x=x_0$
θ angle of the bed slope
ρ density of fluid
$\Delta\rho$ density difference between the two fluids in motion
$\Delta\rho_0$ value of $\Delta\rho$ when the front reaches the position $x=x_0$
$\Delta\rho_*$ density difference between the two fluids before supply
τ_b bed shear stress
τ_i interfacial shear stress