Mutual Inductance between the Semicircular Bus and the Straight Bus (I)

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Abstract

The curved busses are often used and it is specially important to calculate self and mutual inductance of heavy current busses. Therefore in this paper the mutual inductance between the semicircular bus and the straight bus is given.

And the numerical example is shown.

List of symbols

C_1 : straight conductor
C_2 : semicircular conductor
dl_1 : element of the straight conductor
dl_2 : element of the semicircular conductor
l : length of C_1
r : radius of C_2
R : distance between any point P_1 and P_2 selected at random on the conductor C_1 and C_2
θ : angle between the straight conductor and radius for point P_2

1. Introduction

On the secondary busses of the transformers used for the electric furnaces, the currents are heavy and the power factor is affected remarkably by reactance of the busses.

Therefore, attention must be paid to the arrangements of the conductors. In this paper we calculate mutual inductance between the curved bus and the straight bus. Flexible cables of the electric furnaces are necessary because the electrodes are moved up and down by the control device. They are curvilinear.

Assuming that flexible cables are semicircular, we calculate mutual inductance between them and the straight busses.

It is assumed that the conductors are of circular crosssection and that the current is concentrated along the axis of the conductors.

It has reported that little error is introduced by this assumption[1].

2. Arrangements of the busses

In Fig.1 both the semicircular conductor and the straight conductor lie on a same plane. Diameter of the semicircular conductor and the center-line of the straight conductor are in the same line.

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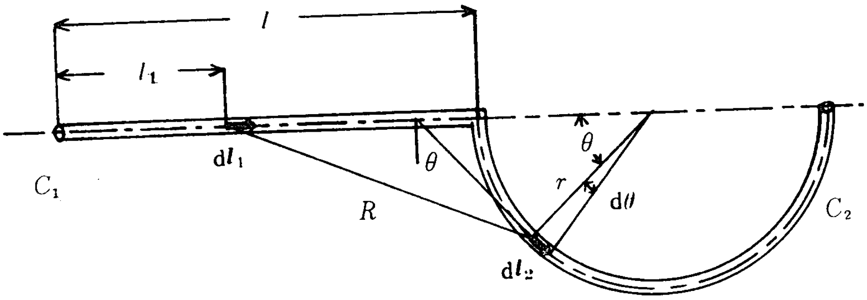

Fig. 1 Arrangement of the conductors

3. Calculation of mutual inductance

In Fig.1

$$R^2=(r\sin\theta)^2+(r-r\cos\theta+l-l_1)^2$$

Therefore

$$R=\sqrt{(r\sin\theta)^2+(r\cos\theta+l-l_1)^2} \tag{1}$$

And moreover scalar product $dl_1 \cdot dl_2$ is

$$dl_1\cdot dl_2=dl_1\,dl_2\cos\left(\frac{\pi}{2}-\theta\right)=r\sin\theta\,d\theta\,dl_1 \tag{2}$$

By Neumann's formula, mutual inductance M between the semicircular bus and the straight bus in Fig.1 is

$$M=\frac{\mu_0}{4\pi}\int\int\frac{dl_1\cdot dl_2}{R}$$

$$=\frac{\mu_0 r}{4\pi}\int_{\theta=0}^{\theta=\pi}\int_{l_1=0}^{l_1=l}\frac{\sin\theta\,d\theta\,dl_1}{\sqrt{(r\sin\theta)^2+(r-r\cos\theta+l-l_1)^2}} \tag{3}$$

First, we substitute

$$\Big|_0^l\ r-r\cos\theta+l-l_1=x\ \Big|_{r-r\cos\theta+l}^{r-r\cos\theta}\quad,\quad dl_1=-dx \tag{4}$$

And first integrating with l_1

$$\int_0^l\frac{dl_1}{\sqrt{(r\sin\theta)^2+(r-r\cos\theta+l-l_1)^2}}$$

$$=\int_{r-r\cos\theta+l}^{r-r\cos\theta}\frac{-dx}{\sqrt{x^2+r^2\sin^2\theta}}$$

$$=-\Big[\log_e(x+\sqrt{x^2+r^2\sin^2\theta})\Big]_{r-r\cos\theta+l}^{r-r\cos\theta}$$

$$=\log_e\{r+l-r\cos\theta+\sqrt{(r+l-r\cos\theta)^2+r^2\sin^2\theta}\}$$

$$-\log_e\{r-r\cos\theta+\sqrt{(r-r\cos\theta)^2+r^2\sin^2\theta}\} \tag{5}$$

Using Equation (5), Equation (3) becomes

$$M=\frac{\mu_0 r}{4\pi}\int_0^\pi\sin\theta\log_e\{r+l-r\cos\theta+\sqrt{(r+l-r\cos\theta)^2+r^2\sin^2\theta}\}d\theta$$

$$-\frac{\mu_0 r}{4\pi}\int_0^\pi\sin\theta\log_e\{r-r\cos\theta+\sqrt{(r-r\cos\theta)^2+r^2\sin^2\theta}\}d\theta$$

$$=M_1-M_2 \tag{6}$$

where we put the first and second term of Equation (6) into M_1, M_2 respectively.

First we calculate M_1.

Substituting $\cos\theta$ for t and integrating with t

$$M_1=\frac{\mu_0 r}{4\pi}\int_{-1}^{1}\log_e\{r+l-rt+\sqrt{(r+l-rt)^2+r^2(1-t^2)}\}dt$$

$$=\frac{\mu_0 r}{4\pi}\int_{-1}^{1}\log_\varepsilon\{r+l-rt+\sqrt{(r+l)^2+r^2-2r(r+l)t}\}dt \tag{7}$$

Again substituting next

$$\left.\begin{matrix}1\\-1\end{matrix}\right| (r+l)^2+r^2-2r(r+l)t=T^2 \left|\begin{matrix}l^2\\(l+2r)^2\end{matrix}\right., \quad -r(r+l)dt=TdT$$

And using partial integration, we calculate M_1.

$$M_1=\frac{\mu_0 r}{4\pi}\int_l^{l+2r}\log_\varepsilon\left\{\frac{T^2+(2r+2l)T+l^2+2lr}{2r+2l}\right\}\frac{TdT}{r(r+l)}$$

$$=\frac{\mu_0 r}{4\pi}\left[\frac{T^2}{2r(r+l)}\log_\varepsilon\left\{\frac{T^2+(2r+2l)T+l^2+2lr}{2r+2l}\right\}\right]_l^{2r+l}$$

$$-\frac{\mu_0 r}{4\pi}\int_l^{2r+l}\frac{T^3+(l+r)T^2}{T^2+(2r+2l)T+l^2+2lr}dT$$

$$=\frac{\mu_0 r}{4\pi}\left[\frac{T^2}{2r(r+l)}\log_\varepsilon\frac{T^2+(2r+2l)T+l^2+2lr}{2(r+l)}\right]_l^{2r+l}$$

$$-\frac{\mu_0 r}{4\pi(l+r)}\left[\frac{T^2}{2}-(r+l)T+\frac{1}{2}(2r^2+2rl+l^2)\log_\varepsilon(T+l)(T+l+2r)-(r+l)\log_\varepsilon\frac{T+l}{T+l+2r}\right]_l^{2r+l}$$

$$=\frac{\mu_0 r}{4\pi}\left\{\frac{(l+2r)^2}{2r(r+l)}\log_\varepsilon 2(l+2r)-\frac{l^2}{2r(r+l)}\log_\varepsilon 2l\right\}$$

$$-\frac{\mu_0 r}{4\pi}\left\{\frac{2r^2+2rl+l^2}{2r(r+l)}\log_\varepsilon\frac{l+2r}{l}-\log_\varepsilon\frac{(r+l)^2}{l(l+2r)}\right\}$$

$$=\frac{\mu_0 r}{2\pi}\log_\varepsilon 2(l+r) \tag{8}$$

Next we calculate M_2.

Substituting similarly in the cace M_1.

$$M_2=\frac{\mu_0 r}{4\pi}\int_{-1}^{1}\log_\varepsilon\{r-rt+\sqrt{(r-rt)^2+r^2(1-t^2)}\}dt$$

$$=\frac{\mu_0 r}{4\pi}\left\{2\log_\varepsilon r+\int_{-1}^{1}\log_\varepsilon\sqrt{1-t}\,dt+\int_{-1}^{1}\log_\varepsilon(\sqrt{1-t}+\sqrt{2})dt\right\} \tag{9}$$

where we put the second and third term of Equation(9) into M_{21}, M_{22} respectively. And first substituting $1-t$ for T and calculating M_{21}

$$M_{21}=\frac{\mu_0 r}{4\pi}\int_{-1}^{1}\text{loe}_\varepsilon\sqrt{1-t}\,dt$$

$$=\frac{\mu_0 r}{4\pi}\int_0^2\frac{1}{2}\log_\varepsilon T\cdot dT$$

$$=\frac{\mu_0 r}{4\pi}\left[\frac{T}{2}\log_\varepsilon T-\frac{T}{2}\right]_0^2$$

$$=\frac{\mu_0 r}{4\pi}\left\{\log_\varepsilon 2-1-\frac{1}{2}\lim_{T\to 0}(T\log_\varepsilon T-T)\right\}$$

$$=\frac{\mu_0 r}{4\pi}\left(\log_\varepsilon 2-1+\frac{1}{2}\lim_{T\to 0}T\right)=\frac{\mu_0 r}{4\pi}(\log_\varepsilon 2-1) \tag{10}$$

Next calculating M_{22}

$$M_{22}=\frac{\mu_0 r}{4\pi}\int_0^{\sqrt{2}}\log_\varepsilon(T+\sqrt{2})\cdot 2TdT \quad (\text{where}\sqrt{1-t}=T)$$

$$=\frac{\mu_0 r}{4\pi}\left[(T^2-2)\log_\varepsilon(T+\sqrt{2})-\frac{T^2}{2}+\sqrt{2}\,T\right]_0^{\sqrt{2}}$$

$$=\frac{\mu_0 r}{4\pi}(1+\log_\varepsilon 2) \tag{11}$$

Inserting Equation(10) and Equation(11) into Equation(9) we obtain

$$M_2=\frac{\mu_0 r}{4\pi}(2\log_\varepsilon r+\log_\varepsilon 2-1+\log_\varepsilon 2+1)$$

$$=\frac{\mu_0 r}{4\pi}(2\log_\varepsilon 2r)=\frac{\mu_0 r}{2\pi}\log_\varepsilon 2r \tag{12}$$

Moreover inserting Equation(8) and Equation(12) into Equation(6) and we obtain M finaly

$$M=\frac{\mu_0 r}{2\pi}\{\log_\varepsilon 2(l+r)-\log_\varepsilon 2r\}$$

$$=\frac{\mu_0 r}{2\pi}\log_\varepsilon\frac{r+l}{r} \quad (13)$$

4. Numerical example

Fig.2 is relationship between M and $u(=l/r)$ for the case that l is parameter. It shows that M decreases as r decreases, where l is constant.

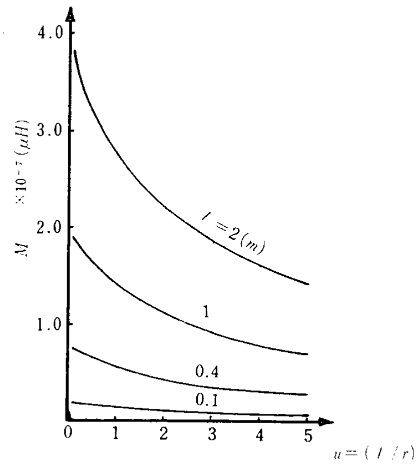

Fig.2 Relationship between M and u

5. Conclusion

The result is simple. It is useful to design arrangements of the heavy current busses.

Reference

1) T. MOMOTA Researches of the Electrotechnical Laboratory. No. 503 (1949)

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