LETTER

Surface Waves along a Boundary of Single Negative Material

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SUMMARY Existence of a surface wave along the boundary between the semi-infinite materials, one of which is a free-space and the other is a material with either negative permeability or negative permittivity, is theoretically investigated. Surface waves exist in only limited combination of negative and positive signs of the material parameters. In addition, by analyzing the surface wave in a finite-thickness slab with negative permeability, its mode profile has been obtained for two different types of symmetry. From these results, the present paper predicts the possibility of a surface wave directional coupler based on a single slab transmission along its top and bottom surfaces.

key words: *surface wave, metamaterial, negative permeability, negative permittivity, directional coupler*

1. Introduction

Metamaterials with negative permeability and/or permittivity have received much attention from many researchers due to their extraordinary electromagnetic (EM) characteristics [1]–[5]. It has recently been reported that negative permittivity and permeability can be constructed by arranging split-ring resonators [2] and by arranging thin metal wires [3], [4] in the host media, respectively.

It has already been known that the negative permittivity or permeability can be observed in a conducting material [6] or magnetized ferrite with a tensor permeability [7]. The surface plasmon, impedance surface wave [8], and magnetostatic surface wave (MSSW) are some examples propagating on the negative permittivity or permeability materials (single negative material) boundary. But, in the case of MSSW propagation, the off-diagonal term of permeability tensor plays an important role. Meanwhile, our study will show TM and TE surface waves exist on the boundary of material with scalar negative permittivity and permeability, and thus tries to clarify the principle of existence of surface wave at first, coping with the new application of metamaterials.

Secondly, we will also show that surface waves propagate in the finite-thickness slab with negative permeability from a practical point of view. We will obtain the even- and odd-symmetric surface waves theoretically. If one prepares two slab waveguides of positive permittivity, a directional coupler will be fabricated keeping a proper distance between them. However, according to our theoretical analysis, a surface wave directional coupler could be constructed with an only single slab structure of negative permeability whose top and bottom surfaces work as the wave-guiding system separately.

Double-layered structure with a pair of single-negative, double-negative, and/or double-positive layers has been studied [9], [10]. Surface waves along single-layered structure of double-negative (left-handed) material were also demonstrated [11]. However, the slab structure, treated in our study, has only one layer with single-negative material. In other words, our study claims that the simplest structure of single-negative material can support a surface wave.

2. Surface Wave along Semi-Infinite Material Boundary

2.1 Basic Structure

As shown in Fig. 1, a semi-infinite material with its permeability μ or permittivity ε is negative in Region II, is faced at the $x = 0$ surface with a semi-infinite free-space whose permittivity ε_0 and permeability μ_0 is both positive real in Region I. The coordinate system is also shown in the same figure, where the structure has no variation in the y- and z-directions. It is assumed that the time and z-dependence of the EM fields are $\exp\{j(\omega t - \beta z)\}$.

We have tried to find the TE and TM surface waves for all possible combinations of positive and negative material parameters in Region II. The results are shown in Table 1. The surface wave can exist only for a limited material parameters' sign combination. Next, we will show the field distribution for each possible combination of the materials.

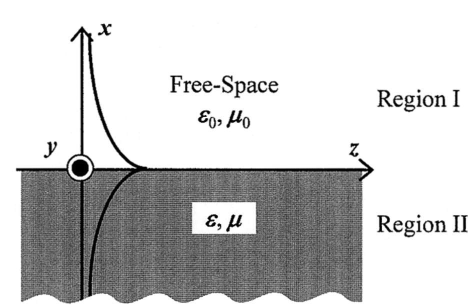

Fig. 1 Semi-infinite single-negative material and surface wave along it.

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Table 1 Existence of surface wave for material parameter combinations.

	$\varepsilon > 0, \mu < 0$	$\varepsilon < 0, \mu > 0$
TE wave ($E_z = 0$)	○	×
TM wave ($H_z = 0$)	×	○

2.2 TE Surface Wave

The TE surface wave with $E_z = 0$ can exist only for the combination of material parameters in Region II of positive permittivity $\varepsilon = \varepsilon_1$ and negative permeability $\mu = -\mu_1$, where ε_1 and μ_1 are the positive real constants. The field components can be shown in the following expressions.

For Region I ($x \geq 0$), setting $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$

$$H_x = -\frac{j\beta A}{\delta_0} e^{-\delta_0 x},\ H_z = A e^{-\delta_0 x},\ E_y = \frac{j\omega\mu_0 A}{\delta_0} e^{-\delta_0 x}, \tag{1}$$

where $\delta_0 = \sqrt{\beta^2 - \omega^2 \varepsilon_0 \mu_0}$ and A is an arbitrary real coefficient.

For Region II ($x \leq 0$), setting $\varepsilon = \varepsilon_1$ and $\mu = -\mu_1$,

$$H_x = \frac{j\beta A_1}{\delta_1} e^{\delta_1 x},\ H_z = A_1 e^{\delta_1 x},\ E_y = \frac{j\omega\mu_1 A_1}{\delta_1} e^{\delta_1 x}, \tag{2}$$

where $\delta_1 = \sqrt{\beta^2 + \omega^2 \varepsilon_1 \mu_1}$ and A_1 is an arbitrary real coefficient.

By applying the boundary conditions at the $x = 0$ surface for transverse electric and magnetic field, H_z and E_y, the following eigenvalue relationship has been obtained,

$$\mu_0 \delta_1 = \mu_1 \delta_0,\ A = A_1. \tag{3}$$

Then, the propagation constant β can be expressed as follows,

$$\beta^2 = \frac{\omega^2 \mu_0 \mu_1 (\varepsilon_1 \mu_0 + \varepsilon_0 \mu_1)}{\mu_1^2 - \mu_0^2}. \tag{4}$$

2.3 TM Surface Wave

The TM surface wave with $H_z = 0$ can exist only for the combination of material parameters in Region II of negative permittivity $\varepsilon = -\varepsilon_1$ and positive permeability $\mu = \mu_1$. Taking account of the duality in the electromagnetics and applying the boundary condition at the $x = 0$ surface, the expressions corresponding to Eqs. (1)–(4) for TM surface wave can be obtained as follows,

$$E_x = -\frac{j\beta A}{\delta_0} e^{-\delta_0 x},\ E_z = A e^{-\delta_0 x},\ H_y = -\frac{j\omega\varepsilon_0 A}{\delta_0} e^{-\delta_0 x},\quad \text{for } x \geq 0. \tag{5}$$

$$E_x = \frac{j\beta A}{\delta_1} e^{\delta_1 x},\ E_z = A e^{\delta_1 x},\ H_y = -\frac{j\omega\varepsilon_1 A}{\delta_1} e^{\delta_1 x},\quad \text{for } x \leq 0. \tag{6}$$

$$\varepsilon_0 \delta_1 = \varepsilon_1 \delta_0, \tag{7}$$

$$\beta^2 = \frac{\omega^2 \varepsilon_0 \varepsilon_1 (\varepsilon_1 \mu_0 + \varepsilon_0 \mu_1)}{\varepsilon_1^2 - \varepsilon_0^2}. \tag{8}$$

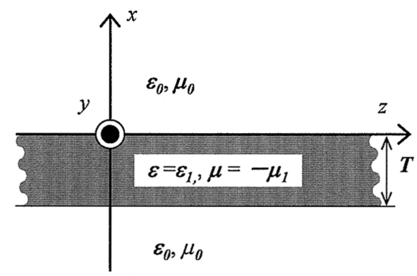

Fig. 2 Finite-thickness slab with negative permeability.

3. Surface Wave in Finite-Thickness Slab with Negative Permeability

3.1 Field Expression and Eigenvalue Equation

Because the semi-infinite material can not be realized for the practical manufacturing, we analyze the TE surface wave in the finite-thickness slab with negative permeability where slab thickness is T as shown in Fig. 2. The coordinate system is also shown in the same figure. Then, the slab region, that is $-T \leq x \leq 0$, is constructed with the positive permittivity $\varepsilon = \varepsilon_1$ and negative permeability $\mu = -\mu_1$ material, and the upper and lower regions of the slab, that is $x \geq 0$ and $x \leq -T$, are the free-space with the material parameter ε_0 and μ_0. The EM field components of this surface wave can be expressed as follows,

$$\begin{cases} H_x = -\dfrac{j\beta}{\delta_0} e^{-\delta_0 x} \\ H_z = e^{-\delta_0 x} & x \geq 0, \\ E_y = -\dfrac{j\omega\mu_0}{\delta_0} e^{-\delta_0 x} \end{cases}$$
$$\begin{cases} H_x = \dfrac{j\beta}{\delta_1} \left(-A_1 e^{-\delta_1 x} + B_1 e^{\delta_1 x}\right) \\ H_z = A_1 e^{-\delta_1 x} + B_1 e^{\delta_1 x} & -T \leq x \leq 0, \\ E_y = \dfrac{j\omega\mu_1}{\delta_1} \left(-A_1 e^{-\delta_1 x} + B_1 e^{\delta_1 x}\right) \end{cases}$$
$$\begin{cases} H_x = -\dfrac{j\beta A_2}{\delta_0} e^{\delta_0 x} \\ H_z = A_2 e^{\delta_0 x} & -T \geq x, \\ E_y = -\dfrac{j\omega\mu_0 A_2}{\delta_0} e^{\delta_0 x} \end{cases} \tag{9}$$

where δ_0 and δ_1 are the same quantities appeared in Eqs. (1) and (2), and A_1, B_1, and A_2 are arbitrary coefficients. By applying the boundary conditions on the surface at $x = 0$ and $x = -T$, and eliminating the coefficients A_1, B_1, and A_2, the following eigenvalue equation for the surface wave can be obtained

$$(\mu_1 \delta_0 - \mu_0 \delta_1)^2 = (\mu_1 \delta_0 + \mu_0 \delta_1)^2 e^{-2\delta_1 T}. \tag{10}$$

3.2 Surface Waves along the Negative Permeability Slab

Solving Eq. (10) numerically, we can obtain the eigenvalues

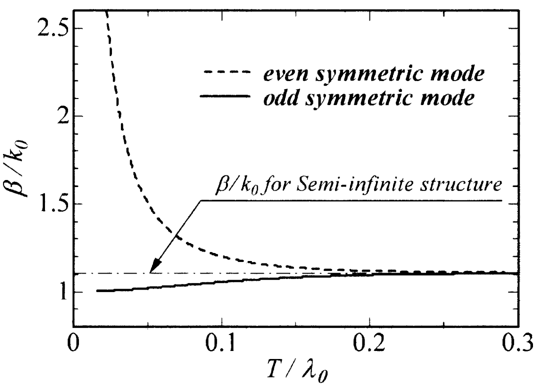

Fig. 3 The normalized propagation constant as a function of normalized slab thickness, where $\varepsilon/\varepsilon_0 = 1.144$ and $\mu/\mu_0 = -6.071$, respectively, at the fixed operating frequency 13.6 GHz.

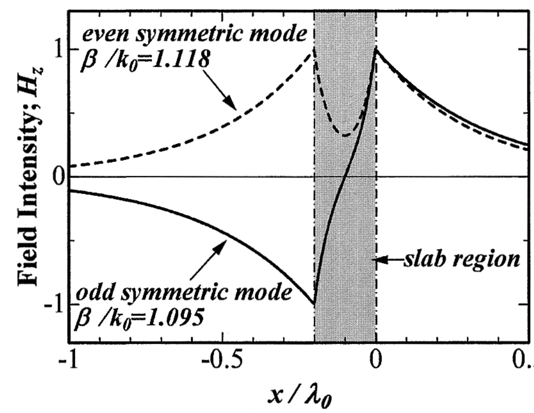

Fig. 4 The mode profiles of the longitudinal field component, H_z, for each surface-wave at the normalized slab thickness $T/\lambda_0 = 0.2$. The other parameters are the same as those in Fig. 3.

for the surface waves. The negative permeability could be realized by stacking the cubic cell with printed double splitring resonators on its walls [2]. In this structure, the permeability becomes negative at some frequency. According to the reference [2] with the resistivity $\sigma = 10.0\,\Omega$, at the fixed operating frequency 13.6 GHz, the permittivity and the permeability of the material is estimated as $\varepsilon/\varepsilon_0 = 1.144$ and $\mu/\mu_0 = -6.071$, respectively. In the following simulations, these values are used for the analysis.

Figure 3 presents the numerical results for the normalized propagation constant β/k_0 as a function of normalized slab thickness T/λ_0, where k_0 and λ_0 are the wave-number and the wavelength in free-space, respectively. From this result, two surface waves with different propagation constants are observed at any slab thickness. The normalized propagation constant for the semi-infinite structure presented in Eq. (4) with the same numerical parameters is also drawn in the same figure with the dotted dash line. As the slab thickness becomes larger, the propagation constant of the two surface waves approach to that of the semi-infinite structure. From a different point of view, it is also verified by the fact that Eq. (3) for the semi-infinite structure becomes equivalent with Eq. (10) when slab thickness T approaches to infinity.

Next, we have estimated the field profiles of each surface wave. The result shown in Fig. 4 tells that each wave has the even or odd symmetric profile with respect to the center of the slab, reflecting the symmetry of the structure. It should also be noted that there is no thickness mode as is the case for a normal dielectric slab of positive permittivity.

For the negative permittivity slab, on the other hand, due to the duality of the electric and magnetic fields, it is easily demonstrated that a TM surface mode propagates with proper interchange of the electric and magnetic field components in the similar manner with Section 2.3.

Changing the subject to applications, a surface wave directional coupler consisting of two YIG films separated by a dielectric sheet has been proposed [12]. Their directional coupler with double YIG films has very similar characteristics of propagation constant and field profiles with those for our single slab structure of negative permeability. Compare their Fig. 2 in the reference [12] with our Figs. 3 and 4. In our structure, one boundary surface confines the EM field, and thus the surface wave directional coupler can be constructed where its top and bottom surfaces of the slab exchanges energy as the wave-guiding regions. A much detailed property of the new coupler will be presented in the near future.

4. Conclusions

It is theoretically presented that the existence of a surface wave along the boundary between the semi-infinite materials, one of which is a free-space and the other is a material of either negative permeability or negative permittivity. TE and TM type surface waves exist in very simple combination of negative and positive signs of the material parameters. In addition, by analyzing the surface wave in the finite-thickness slab with negative permeability, its mode profile has been obtained for two different types of symmetry. The possibility of a surface wave directional coupler based on a single slab transmission along its top and bottom surfaces is presented.

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